Instructor's Manual for

Industrial Organization
Theory and Applications

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Contents

To the Instructor

Before planning the course, I urge the instructor to read carefully the Preface of the book that suggests different ways of organizing courses for different levels of students and also provides a list of calculus-free topics.

The goals of this manual are:

- To provide the instructor with my solutions for all the problems listed at the end of each chapter;
- to convey to the instructor my views of what the important concepts in each topic are;
- to suggest which topics to choose for different types of classes and levels of students.

Finally, please alert me to any errors or incorrect presentations that you detect in the book and in this manual. (e-mail addresses are given below). Note that the errata files (according to the printing sequence) are posted on the Web in PDF format. Before reporting errors and typos, please view the errata files to see whether the error you found was already identified and corrected.

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Chapter 2

Basic Concepts in Noncooperative Game Theory

An instructor of a short course should limit the discussion of game theory to the four most important concepts in this chapter that are essential for the understanding most of the analyses presented in this book:

1. The definition of a game (Definition 2.1): It is important that the student will understand that a game is not properly defined unless the list of players, the action set of each player, and the payoff functions are clearly stated. It is important that the student will understand the meaning of the term *outcome* as a list of the specific actions chosen by each player (and not a list of payoffs as commonly assumed by students).
2. Nash equilibrium (Definition 2.4).
3. Welfare comparisons among outcomes (Definition 2.6).
4. Extensive form games, strategies (compare with actions), subgames, and the SPE (Definition 2.10).

I urge the instructor to discuss the issues of existence, uniqueness, and multiple equilibria in class. More precisely, it is important that the student will know that in order to prove existence, it is sufficient to find only one NE outcome; however, to prove nonexistence, the student must go over all outcomes and show that at least one player benefits from unilateral deviation.

I believe that the above can be covered in 2 lectures, or in three hours of instruction. If you wish to devote more time to game theory, you can introduce the equilibrium in dominant actions (Definition 2.3) before teaching the NE equilibrium concept. If you wish to emphasis more game theory, I advise covering repeated games (section 2.3).

Answers to Exercises

1. (a) It is straightforward to conclude that

$$R^1(a^2) = \begin{cases} \text{WAR} & \text{if } a^2 = \text{WAR} \\ \text{WAR} & \text{if } a^2 = \text{PEACE} \end{cases} \quad \text{and} \quad R^2(a^1) = \begin{cases} \text{WAR} & \text{if } a^1 = \text{WAR} \\ \text{WAR} & \text{if } a^2 = \text{PEACE.} \end{cases}$$

That is, WAR is each player's best response to each action taken by the other player (hence, WAR is a dominant action for each player). Now, $(\hat{a}^1, \hat{a}^2) = (\text{WAR}, \text{WAR})$ is a (unique) NE since this outcome is on the best-response function of each player.

(b)

$$R^J(a^R) = \begin{cases} \omega & \text{if } a^R = \omega \\ \phi & \text{if } a^R = \phi \end{cases} \quad \text{and} \quad R^R(a^J) = \begin{cases} \phi & \text{if } a^J = \omega \\ \omega & \text{if } a^J = \phi. \end{cases}$$

There does not exist a NE for this game since there does not exist an outcome the is on both best-response functions. That is, $R^J(\omega) = \omega$, but $R^R(\omega) = \phi$, but $R^J(\phi) = \phi$, but $R^R(\phi) = \omega$, so $R^J(\omega) = \omega$, and so on.

(c)

$$R^\alpha(a^\beta) = \begin{cases} \text{B} & \text{if } a^\beta = \text{L} \\ \text{T} & \text{if } a^\beta = \text{R} \end{cases} \quad \text{and} \quad R^\beta(a^\alpha) = \begin{cases} \text{L} & \text{if } a^\alpha = \text{T} \\ \text{R} & \text{if } a^\alpha = \text{B}. \end{cases}$$

A NE does not exist for this game since $R^\alpha(L) = B$, but $R^\beta(B) = R$, but $R^\alpha(R) = T$, so $R^\beta(T) = L$, and so on.

2. (a) If (T, L) is a NE, then

$$\pi^\alpha(T,L) = a \geq e = \pi^\alpha(B,L), \text{ and } \pi^\beta(T,L) = b \geq d = \pi^\beta(T,R).$$

(b) If (T, L) is an equilibrium in dominant actions, then T has to be a dominant action for player α, that is

$$\pi^\alpha(T,L) = a \geq e = \pi^\alpha(B,L) \text{ and } \pi^\alpha(T,R) = c \geq g = \pi^\alpha(B,R);$$

and L is a dominant action for player β, that is

$$\pi^\beta(T,L) = b \geq d = \pi^\beta(T,R), \text{ and } \pi^\beta(B,L) = f \geq h = \pi^\beta(T,R).$$

Let us observe that the parameter restrictions given in part (a) are also included in part (b) confirming Proposition 2.1 which states that an equilibrium in dominant actions is also a NE.

(c) i. (T, L) Pareto dominates (T, R) if ($a \geq c$ and $b > d$) or ($a > c$ and $b \geq d$);
ii. (T, L) Pareto dominates (B, R) if ($a \geq g$ and $b > h$) or ($a > g$ and $b \geq h$);
iii. (T, L) Pareto dominates (B, L) if ($a \geq e$ and $b > f$) or ($a > e$ and $b \geq f$).

(d) Loosely speaking, the outcomes are Pareto noncomparable if one player prefers (T, L) over (B, R), whereas the other player prefers (B, R) over the outcome (T, L). Formally, either

$$\pi^\alpha(T,L) = a > g = \pi^\alpha(B,R) \text{ but } \pi^\beta(T,L) = b < h = \pi^\beta(B,R),$$

or

$$\pi^\alpha(T,L) = a < g = \pi^\alpha(B,R) \text{ but } \pi^\beta(T,L) = b > h = \pi^\beta(B,R).$$

3. (a) There are three Pareto optimal outcomes:
i. $(n_1, n_2) = (100, 100)$, where $\pi^1(100, 100) = \pi^2(100, 100) = 100$;
ii. $(n_1, n_2) = (100, 99)$, where $\pi^1(100, 99) = 98$ and $\pi^2(100, 99) = 101$;
iii. $(n_1, n_2) = (99, 100)$, where $\pi^1(99, 100) = 101$ and $\pi^2(99, 100) = 98$;

(b) There is no NE for this game.[1] To prove this, we have to show that for every outcome (n_1, n_2), one of the players will benefit from changing his or her declared value. Let us look at the following outcomes:

[1] A typo in the question (first printing) leads to this undesirable result (see Basu, K. 1994. "The Traveler's Dilemma: Paradoxes of Rationality in Game Theory." *American Economic Review* 84: 391–395; for a mechanism that generates $(2, 2)$ as a unique NE outcome).

i. If $(n_1, n_2) = (100, 100)$, then player 1 can increase his or her payoff to $\pi^1 = 101$ by declaring $n_1 = 99$.
ii. If $(n_1, n_2) = (99, 100)$, then player 2 can increase his or her payoff from $\pi^2 = 98$ to $\pi^2 = 99$ by declaring $n_2 = 99$. Hence, players always benefit from declaring a value of one dollar lower than the other player, and so on.
iii. If $(n_1, n_2) = (2, 2)$, then player 1 can increase his or her payoff from $\pi^1 = 2$ to $\pi^1 = 98$ by declaring $n_1 = 100$.

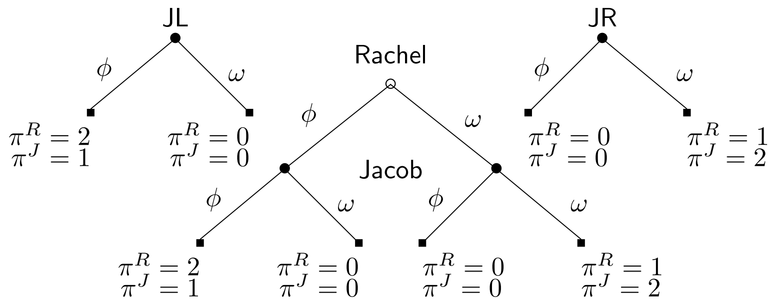
4. (a) There are several NE outcomes for this game. For example, (LG, SM, SM) is a NE outcome since

$$\begin{aligned}
\pi^C(LG, SM, SM) = \alpha &\geq \gamma = \pi^C(SM, SM, SM) \\
\pi^F(LG, SM, SM) = \beta &= \beta = \pi^F(LG, LG, SM) \\
\pi^G(LG, SM, SM) = \beta &= \beta = \pi^G(LG, SM, LG).
\end{aligned}$$

Hence, no firm finds it profitable to unilaterally change the type of cars it produces.

(b) Again, there are several NE. The outcome (LG, SM, SM) is also a NE for this game, and the proof is identical to the one given in part (a).

5. (a) There are three subgames: the game itself, and two proper subgames labeled JL (for Jacob left) and JR (for Jacob right). The three subgames are illustrated in Solution-Figure 2.1.



Solution-Figure 2.1: Three subgames of the dynamic Battle of the Sexes

(b) One easy way to find NE *outcomes* for an extensive form game is to construct a normal-form representation.[2] However, the normal-form representation is already given in Table 2.2. Also, equation (2.1) proves that the NE outcomes involve the two players going *together* either to football or to the opera. Formally, it is easy to verify that the following *three* outcomes constitute NE:

$$s^J = \begin{cases} \phi & \text{if } s^R = \omega \\ \phi & \text{if } s^R = \phi \end{cases} \quad \text{and} \quad s^R = \phi,$$

[2]This subquestion may confuse students who are already thinking in terms of backward induction. Since our analysis is based on SPE you may want to avoid assigning this subquestion (only). Also, an instructor teaching from the first printing is urged to change this subquestion to finding the NE for the entire game only; hence, to postpone finding the NE for the subgames to the next subquestion, where the NE of the subgames are used to find the SPE.

$$s^J = \begin{cases} \omega & \text{if } s^R = \omega \\ \omega & \text{if } s^R = \phi \end{cases} \quad \text{and} \quad s^R = \omega,$$
$$s^J = \begin{cases} \omega & \text{if } s^R = \omega \\ \phi & \text{if } s^R = \phi \end{cases} \quad \text{and} \quad s^R = \phi. \tag{2.1}$$

(c) Using backward induction, we first construct Jacob's strategy (which constitutes the Nash equilibria for the two proper subgames). Looking at the proper subgames in Solution-Figure 2.1, we conclude that

$$R^J(a^R) = \begin{cases} \omega & \text{if } a^R = \omega \ \text{(subgame JR)} \\ \phi & \text{if } a^R = \phi \ \text{(subgame JL).} \end{cases}$$

Now, we look for Rachel's strategy (given Jacob's best response). If Rachel plays $a^R = \omega$, then her utility is given by $\pi^R(\omega, R^J(\omega)) = 1$. In contrast, if Rachel plays $a^R = \phi$, then her utility is given by $\pi^R(\phi, R^J(\phi)) = 2 > 1$. Altogether, the strategies

$$s^R = a^R = \phi \quad \text{and} \quad R^J(a^R) = \begin{cases} \omega & \text{if } a^R = \omega \\ \phi & \text{if } a^R = \phi \end{cases}$$

constitute a unique SPE.

(d) No! Jacob's best response is to play ϕ whenever Rachel plays ϕ. Thus, Jacob's announcement constitutes an *incredible threat*.[3] Clearly, Rachel, who knows Jacob's best-response function in the second stage, should ignore Jacob's announcement, since Jacob, himself, would ignore it if Rachel plays ϕ in the first stage.

6. (a) Jacob's expected payoff is given by:

$$\begin{aligned} \mathsf{E}\pi^J(\theta,\rho) &= \theta\rho \times 2 + \theta(1-\rho)\times 0 + (1-\theta)\rho \times 0 + (1-\theta)(1-\rho)\times 1 \\ &= 2 + 3\theta\rho - \rho - \theta. \end{aligned}$$

Rachel's expected payoff is given by:

$$\begin{aligned} \mathsf{E}\pi^R(\theta,\rho) &= \theta\rho \times 1 + \theta(1-\rho)\times 0 + (1-\theta)\rho \times 0 + (1-\theta)(1-\rho)\times 2 \\ &= 2 + 3\theta\rho - 2\rho - 2\theta. \end{aligned}$$

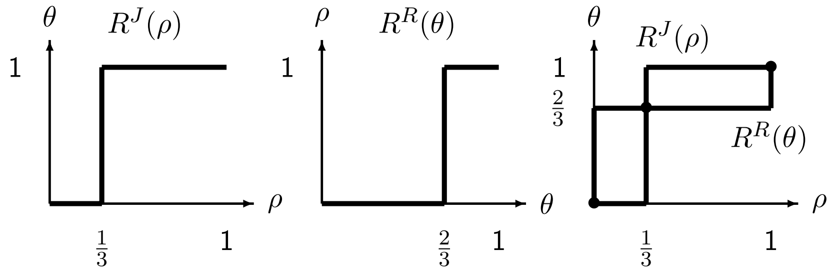
(b) The players' best-response functions are given by

$$R^J(\rho) = \begin{cases} 0 & \text{if } \rho < 1/3 \\ [0,1] & \text{if } \rho = 1/3 \\ 1 & \text{if } \lambda > 1/3 \end{cases} \quad \text{and} \quad R^R(\theta) = \begin{cases} 0 & \text{if } \theta < 2/3 \\ [0,1] & \text{if } \theta = 2/3 \\ 1 & \text{if } \theta > 2/3. \end{cases}$$

The players' best-response functions are drawn in Solution-Figure 2.2.

(c) Solution-Figure 2.2 demonstrates that $(\theta,\rho) = (2/3, 1/3)$ is a NE in mixed actions. Also, note that the outcomes $(0,0)$ and $(1,1)$ are also NE outcomes in mixed actions and are the same as the pure NE outcomes.

[3] Instructors: Here is a good opportunity to discuss the concepts of credible and incredible threats.



Solution-Figure 2.2: Best-response functions for the Battle of the Sexes in mixed actions

(d) Substituting $(\theta, \rho) = (2/3, 1/3)$ into the players' payoff functions (defined above) yield

$$\mathsf{E}\pi^J(2/3, 1/3) = \mathsf{E}\pi^R(2/3, 1/3) = \frac{2}{3}.$$

(e) The best-response functions in Solution-Figure 2.2 intersect three times, meaning that in the mixed extension game, there are two equilibria: the two pure NE outcomes and one NE in mixed actions. In contrast, Figure 2.3 has only one intersection since in that game NE in pure strategies does not exist, and therefore the best-response functions intersect only once, at the NE in mixed actions.

Chapter 3

Technology, Production Cost, and Demand

This chapter summarizes the basic microeconomic tools the students need to know prior to taking this class. My advice for the instructor is not to spend time on this chapter but simply assign this chapter (with or without the exercises) as reading in the first class. However, my advice is to return (or refer) to this chapter whenever definitions are needed. For example, when you first encounter a discussion of returns to scale, I urge you to make a formal definition and refer the students to Definition 3.2. Similarly, when addressing elasticity issues, students should be referred to Definition 3.3.

Answers to Exercises

1. (a) Let $\lambda > 1$. Then, by Definition 3.2, the technology exhibits IRS if

$$(\lambda l)^\alpha(\lambda k)^\beta = \lambda^{\alpha+\beta} l^\alpha k^\beta > \lambda l^\alpha k^\beta,$$

which holds when $\alpha + \beta > 1$. Using the same procedure and Definition 3.2, this technology exhibit CRS if $\alpha + \beta = 1$, and DRS if $\alpha + \beta < 1$.

(b) In this technology, the factors are supporting since

$$MP_L(l,k) \equiv \frac{\partial Q}{\partial l} = \alpha l^{\alpha-1} k^\beta,$$

hence,

$$\frac{\partial MP_L(l,k)}{\partial k} = \alpha\beta l^{\alpha-1} k^{\beta-1},$$

which is greater than zero under the assumption that $\alpha, \beta > 0$.

2. (a) Let $\lambda > 1$. Then, by Definition 3.2, the technology exhibits IRS if

$$(\lambda l)^\alpha + (\lambda k)^\alpha = \lambda^\alpha(l^\alpha + k^\alpha) > \lambda(l^\alpha + k^\alpha),$$

which holds if $\alpha > 1$. Similarly, the technology exhibits DRS if $\alpha < 1$, and CRS if $\alpha = 1$.

(b) $MP_L(l,k) = \alpha l^{\alpha-1}$. Hence, $\partial MP_L(l,k)/\partial k = 0$. Therefore, the factors are neither substitutes nor complements.

3. Let $\lambda > 1$. This (quasi-linear) technology exhibits DRS since

$$\lambda l + \sqrt{\lambda k} < \lambda l + \lambda\sqrt{k} = \lambda(l + \sqrt{k}).$$

4. (a)

$$AC(Q) = \frac{TC(Q)}{Q} = \frac{F}{Q} + c, \text{ and } MC(Q) = \frac{\partial TC(Q)}{\partial Q} = c.$$

These functions are drawn in Figure 4.2 in the book.

(b) Clearly, $AC(Q)$ declines with Q. Hence, $AC(Q)$ is minimized when $Q = +\infty$.

(c) Declining average cost function reflects an increasing returns to scale technology.

5. (a) Using Definition 3.3,

$$\eta_p(Q) \equiv \frac{\partial Q(p)}{\partial Q}\frac{p}{Q} = (-1)\frac{p}{Q} = -\frac{99-Q}{Q}.$$

Hence, $\eta_p(Q) = -2$ when $Q = 33$.

(b) From the above, $\eta_p(Q) = -1$ when $Q = 99/2 = 49.5$.

(c) The inverse demand function is given by $p(Q) = 99 - Q$. Therefore, $TR(Q) = p(Q)Q = (99-Q)Q$. Hence, $MR(Q) \equiv \mathrm{d}TR(Q)/\mathrm{d}Q = 99-2Q$, which is a special case of Figure 3.3.

(d) $MR(Q) = 0$ when $Q = 49.5$. That is, the marginal revenue is zero at the output level where the demand elasticity is -1 (unit elasticity).

(e) Using Figure 3.5,

$$CS(33) = \frac{(99-33)66}{2} = 2178, \quad \text{and} \quad CS(66) = \frac{(99-66)33}{2} = 544.5$$

6. (a)

$$p^{\epsilon} = \frac{A}{Q}. \quad \text{Hence,} \quad p = A^{\frac{1}{\epsilon}}Q^{-\frac{1}{\epsilon}}.$$

(b)

$$\eta_p = \frac{\partial Q(p)}{\partial Q}\frac{p}{Q} = A(-\epsilon)p^{-\epsilon-1}\frac{p}{Q} = \frac{A(-\epsilon)p^{-\epsilon}}{Ap^{-\epsilon}} = -\epsilon.$$

Thus, the elasticity is constant in the sense that it does not vary with the quantity consumed.

(c) The demand is elastic if $\eta_p < -1$, hence if $\epsilon > 1$. The demand is inelastic if $-1 < \eta_p \le 0$, hence if $0 \le \epsilon < 1$.

(d) By Proposition 3.3, $MR = p[1 + 1/(-\epsilon)]$. Hence,

$$\frac{p}{MR} = 1 \Big/ 1 + \frac{1}{-\epsilon} = \frac{\epsilon}{\epsilon - 1}$$

which is independent of Q.

Chapter 4

Perfect Competition

A perfectly competitive market is characterized by nonstrategic firms, where firms take the market price as given and decide how much to produce (and, whether to enter, if free entry is allowed). Most students probably had some discussion of perfectly competitive markets in their intermediate microeconomics class. However, given the importance of this market structure, I urge the instructor to devote some time in order to make sure that the students understand what price-taking behavior means.

It is now the right time to emphasize that, in economics, the term *competitive* refers to *price taking behavior* of agents. For example, we generally assume that our consumers are competitive, which means that they do not bargain over prices and take all prices and their income as given. In a competitive market structure, we make a similar assumption about the firms.

You may want to emphasize and discuss the following points:

1. The assumption of price taking behavior has nothing to do with the number of firms in the industry. For example, one could solve for a competitive equilibrium even in the presence of one firm (see an exercise at the end of this chapter). Note that this confusion often arises since certain market structures yield market allocations similar to the competitive allocation when the number of firms increases (see for example subsection 6.1.2 which shows that the Cournot allocation may converge to the competitive allocation when the number of firms increases to infinity).

2. A major reason for studying and using alternative (noncompetitive) market structures stems from the fact that the competitive market structure very often "fails" to explain why concentrated industries are observed.

3. Another major reason for studying alternative market structures stems from the nonexistence of a competitive equilibrium when firms' technologies exhibit IRS.

Finally, note that this chapter does not solve for a competitive equilibrium under decreasing returns to scale technologies for two reasons: (i) other market structures analyzed in this book are also developed mainly for CRS (unit cost) technologies, and (ii) most students are familiar with the DRS from their intermediate microeconomics class. However, the exercise at the end of this chapter deals with a DRS technology.

Answers to Exercises

1. Firm 1 takes p as given and chooses q_1 to

$$\max_{q_1} \pi_1 = pq_1 - wL_1 = p\sqrt{L_1} - wL_1.$$

The first and second order conditions are given by

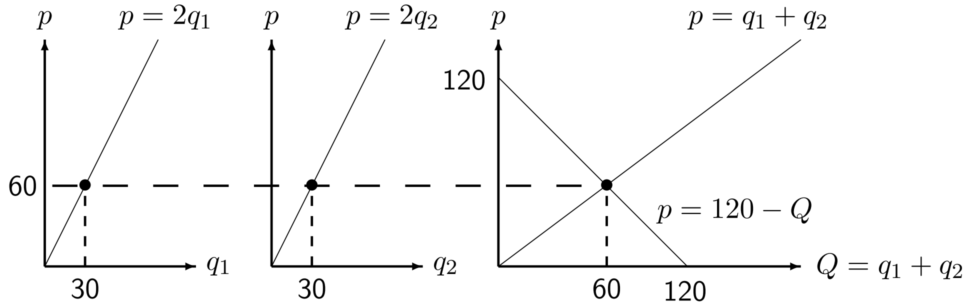
$$0 = \frac{\mathrm{d}\pi_1}{\mathrm{d}L_1} = \frac{p}{2\sqrt{L_1}} - w, \quad \frac{\mathrm{d}^2\pi_1}{\mathrm{d}(L_1)^2} = -\frac{p}{4L_1^{3/2}} < 0.$$

Hence, $q_1 = \sqrt{L_1} = p/(2w)$.

2. Given that there is only one firm, the supply equals demand equilibrium condition yields that $120 - p^e = p^e/2$. Therefore, $p^e = 80$; hence, $q_1^e = Q^e = p^e/2 = 40$.
3. From the production function, we can find the equilibrium employment level to be $L_1^e = (q_1^e)^2 = 1600$. Hence,

$$\pi_1 = p^e q_1^e - wL_1^e = 80 \times 40 - 1600 = 1600.$$

4. Given that the firms have the same technologies, they have the same supply functions. Therefore, the supply equals demand condition becomes $120 - p^e = p^e/2 + p^e/2$, which yields $p^e = 60$, $Q^e = 120 - 60 = 60$, hence, $q_1^e = q_2^e = 30$.
5. Clearly, the competitive price is lower and the aggregate production is higher when the industry consists of two firms.



6.

Solution-Figure 4.1: Competitive equilibrium with two firms

Chapter 5

The Monopoly

Most students encounter the monopoly problem in their intermediate microeconomics class. However, the instructor would probably want to make sure that all students fully understand the monopoly's choice problem and the effect of price elasticity on the monopoly's price, as well as the arguments against monopoly (section 5.2). I also urge the instructor not to skip discussing discriminating monopoly (section 5.3).

The remaining sections, the cartel (section 5.4), and the durable goods monopolies (section 5.5) are more optional depending on the instructor's tastes and students' ability.

Answers to Exercises

1. (a)

$$\eta_p \equiv \frac{\mathrm{d}Q}{\mathrm{d}p}\frac{p}{Q} = a\epsilon p^{-\epsilon-1}\frac{p}{ap^{-\epsilon}} = -\epsilon.$$

Hence, the exponential demand function has a constant price elasticity. By Proposition 3.3,

$$MR(Q) = p\left(1 + \frac{1}{-\epsilon}\right) = p\left(\frac{\epsilon - 1}{\epsilon}\right).$$

(b) Equating marginal revenue to marginal cost yields[1]

$$MR = p^M\left(\frac{\epsilon-1}{\epsilon}\right) = c = MC; \quad \text{hence, } p^M = \frac{\epsilon c}{\epsilon - 1}.$$

(c) As ϵ increases, the demand becomes more elastic, hence, the monopoly price must fall. Formally,

$$\frac{\mathrm{d}p^M}{\mathrm{d}\epsilon} = \frac{c(\epsilon-1) - \epsilon c}{(\epsilon-1)^2} < 0.$$

(d) The first edition (first printing) contains a typo. The question asks what happens to the monopoly's price when $\epsilon \to +1$. Clearly, $p^M \to +\infty$. The reason is, that when $\epsilon = 1$, the (entire) demand has a unit elasticity, implying that revenue does not vary with price (or quantity produced). Hence, given that the revenue is constant (in fact $TR = a$ when $\epsilon = 1$), then the profit maximization problem is reduced to cost minimization which yields that the monopoly would "attempt" to produce as little as possible (but still a strictly positive amount).

[1] The instructor may want to emphasize to the students that in the case of constant-elasticity demand functions (exponential demand functions) it is easier to solve for the monopoly's price first (using Proposition 3.3) and then solve for the quantity produced by substituting the price into the demand function. This procedure becomes very handy when solving monopolistic competition equilibria analyzed in section 7.2.

(e) Inverting the demand function yields $p(Q) = a^{1/\epsilon} Q^{-1/\epsilon}$. Thus,

$$TR(Q) \equiv p(Q)Q = a^{\frac{1}{\epsilon}} Q^{1-\frac{1}{\epsilon}}.$$

Hence,

$$MR(Q) \equiv \frac{\mathrm{d}TR(Q)}{\mathrm{d}Q} = a^{\frac{1}{\epsilon}} Q^{-\frac{1}{\epsilon}} \left(1 - \frac{1}{\epsilon}\right).$$

(f) Equating marginal revenue to marginal cost yields

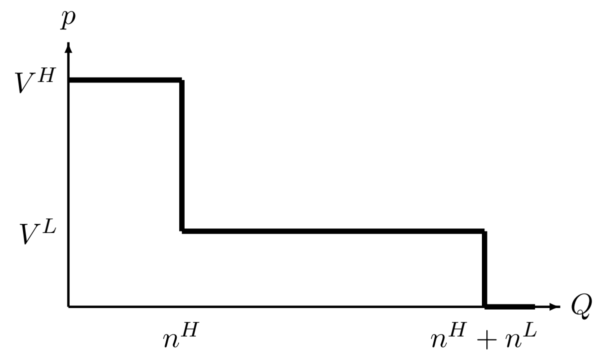
$$a^{\frac{1}{\epsilon}} Q^{-\frac{1}{\epsilon}} \left(1 - \frac{1}{\epsilon}\right) = c,$$

yielding that the monopoly's profit maximizing output is

$$Q^M = a \left(\frac{\epsilon - 1}{\epsilon c}\right)^{\epsilon}.$$

Note that the same result is achieved by substituting p^M (calculated before) into the demand function.

2. (a) Solution-Figure 5.1 illustrates an aggregate demand composed of the two groups of consumers, where each group shares a common valuation for the product.



Solution-Figure 5.1: Aggregate demand composed of two consumer groups

(b) The monopoly has two options[2]: setting a high price, $p = V^H$, or a low price, $p = V^L$. Solution-Figure 5.1 reveals that the profit levels (revenue since production is costless[3]) are given by

$$\pi|_{p=V^H} = n^H V^H, \quad \text{and} \quad \pi|_{p=V^L} = (n^H + n^L)V^L.$$

Comparing the two profit levels yields the monopoly's profit maximizing price. Hence,

$$p^M = \begin{cases} V^H & \text{if } V^H > (n^H + n^L)V^L/n^H \\ V^L & \text{otherwise.} \end{cases}$$

[2]Instructors are urged to assign or discuss this exercise, since it provides a good opportunity to introduce the student to a discrete (logic based) analysis which is used later in a wide variety of topics (see for example the section on tying [section 14.1]).

[3]The first printing of the first edition neglects to assume that production of G-Jeans is costless.

Thus, the monopoly sets a high price if either there are many high valuation consumers (n^H is large) and/or these consumers are willing to pay a very high price (V^H is high).

3. (a) In market 1, $p_1 = 2 - q_1$. Hence, by Proposition 3.2, $MR_1(q_1) = 2 - 2q_1$. Equating $MR_1(q_1) = c = 1$ yields $q_1 = 0.5$. Hence, $p_1 = 1.5$.
   In market 2, $p_2 = 4 - q_2$. Hence, by Proposition 3.2, $MR_2(q_2) = 4 - 2q_2$. Equating $MR_2(q_2) = c = 1$ yields $q_2 = 1.5$. Hence, $p_2 = 2.5$.
   (b) $\pi_1 = (p_1 - c)q_1 = (0.5)^2 = 0.25$, and $\pi_2 = (p_2 - c)q_2 = (1.5)^2 = 2.25$. Summing up, the monopoly's profit under price discrimination is $\pi = 2.5$.
   (c) There are two cases to be considered: (i) The monopoly sets a uniform price $p \geq 2$ thereby selling only in market 2, or (ii) setting $p < 2$, thereby selling a strictly positive amount in both markets. Let us consider these two cases:
      i. If $p \geq 2$, then $q_1 = 0$. Therefore, in this case the monopoly will set q_2 maximize its profit in market 2 only. By subquestion 3a above, $\pi = \pi_2 = 2.25$.
      ii. Here, if $p < 2$, $q_1 > 0$ and $q_2 > 0$. Therefore, aggregate demand is given by $Q(p) = q_1 + q_2 = 2 - p + 4 - p = 6 - 2p$, or $p(Q) = 3 - 0.5Q$. By Proposition 3.2, $MR(Q) = 3 - Q$. Equating $MR(Q) = c = 1$ yields $Q = 2$, hence, $p = 2$. Hence, in this case $\pi = (p - c)2 = 2 < 2.25$.

   Altogether, the monopoly will set a uniform price of $p = 2.5$ and will sell $Q = 1.5$ units in market 2 only.[4]

4. (a)

$$\pi(q_1, q_2) = (100 - q_1/2)q_1 + (100 - q_2)q_2 - (q_1 + q_2)^2.$$

   (b) The two first order conditions are given by

$$0 = \frac{\partial \pi}{\partial q_1} = 100 - q_1 - 2(q_1 + q_2)$$
$$0 = \frac{\partial \pi}{\partial q_2} = 100 - 2q_2 - 2(q_1 + q_2).$$

   Solving for q_1^M and q_2^M yields that $q_1^M = 25$ and $q_2^M = 12.5$.
   (c) Substituting the profit maximizing sales into the market demand functions yield $p_1^M = p_2^M = 87.5$. Hence,

$$\pi(q_1^M, q_2^M) = 87.5 \times 12.5 + 87.5 \times 25 - (12.5 + 25)^2 = 1875.$$

   (d) Now, that each plant sells only in one market, the two first order conditions become

$$0 = \frac{\partial \pi}{\partial q_1} = 100 - q_1 - 2q_1$$
$$0 = \frac{\partial \pi}{\partial q_2} = 100 - 2q_2 - 2q_2.$$

[4] Note that consumers in market 1 are better off under price discrimination than without it, since under no discrimination no output is purchased in market 1. Given that the price in market 2 is the same under price discrimination and without it, we can conclude that in this example, price discrimination is Pareto superior to nonprice discrimination, since both consumer surplus and the monopoly profit are higher under price discrimination.

yielding $q_1^M = 100/3$ and $q_2^M = 25$.

(e) $p_1^M = 100 - 100/6 = 250/3$ and $p_2^M = 100 - 25 = 75$. Hence,

$$\pi = \pi_1(q_1) + \pi_2(q_2) = \frac{250}{3} \times \frac{100}{3} - \left(\frac{100}{3}\right)^2 + 75 \times 25 - 25^2 = 2917.$$

(f) This decomposition increases the monopoly's profit since the technology exhibits DRS.

5. By section 5.3, the discriminating monopoly will set quantities to satisfy $MR_1(q_1^M) = MR_2(q_2^M)$. Hence, by Proposition 3.3

$$MR_1(q_1) = p_1^M \left(1 + \frac{1}{-2}\right) = p_2^M \left(1 + \frac{1}{-4}\right) = MR_2(q_2).$$

Thus, $p_1^M = 1.5 p_2^M$.

6. (a) We solve for the monopoly's profit maximizing prices starting from the second period. The second period outcome may depend on two cases:

First-period consumer does not buy in period 1: Clearly, in this case, the second period profit maximizing price is $p_2 = 50$, yielding a profit level of $\pi_2 = 3 \times 50 = 150$.

First-period consumer buys in period 1: In this case the second period profit maximizing price is again $p_2 = 50$, yielding a profit level of $\pi_2 = 2 \times 50 = 100$.

Altogether, the second-period *price* is independent of the action of the first-period buyer in the first period. Therefore, the maximum price the monopoly can charge the first-period buyer in the first period is $p_1 = 150$.

(b) The second period outcome may depend on two cases:

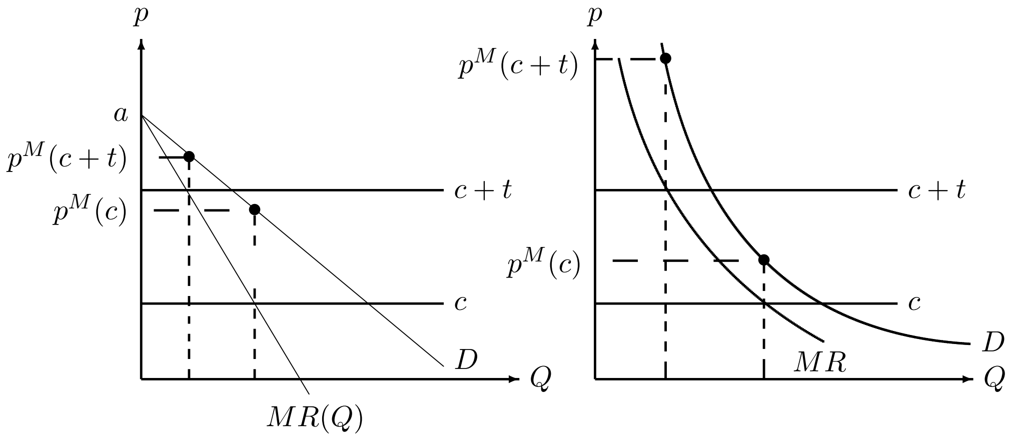
First-period consumer does not buy in period 1: In this case, the monopoly has two choices: (i) charging $p_2 = 20$, and sell to all three consumers, thereby earning a second period profit of $\pi_2 = 3 \times 20 = 60$; or, (ii) charging $p_2 = 50$, and selling only to the second period consumers, thereby earning a second-period profit of $\pi_2 = 2 \times 50 = 100$.

First-period consumer buys in period 1: In this case the second period profit maximizing price is again $p_2 = 50$, yielding a profit level of $\pi_2 = 2 \times 50 = 100$.

Altogether, the second period price is independent of the actions of the first-period buyer. Now, in order to attract the first-period buyer to purchase in period 1, the monopoly should set $p_1 = 40$, thereby extracting all surplus from all consumers.

7. The two cases are illustrated in Solution-Figure 5.2.

(a) Equating marginal revenue to the tax inclusive unit cost yields $a - 2Q = c + t$, or $Q^M = (a - c - t)/2$. Hence, $p^M = (a + c + t)/2$. Now, checking the effect of a tax rate change on the monopoly's price yields that $\mathrm{d}p^M/\mathrm{d}t = 1/2 < 1$. Hence, as illustrated in Solution-Figure 5.2 (left), an increase in t raises the monopoly price by less than t.



Solution-Figure 5.2: How the monopoly price varies with a specific tax

(b) Using Proposition 3.3,

$$MR = p^M \left(1 + \frac{1}{-2}\right) = c + t, \quad \text{yielding} \quad p^M = 2(c+t).$$

Now, checking the effect of a tax rate change on the monopoly's price yields that $\mathrm{d}p^M/\mathrm{d}t = 2 > 1$. Hence, as illustrated in Solution-Figure 5.2 (right), an increase in t raises the monopoly price by more than t.

Chapter 6

Markets for Homogeneous Products

The Cournot, Bertrand, and sequential moves market structures are essential for a further study of industrial organization.

The important issues to discuss and emphasize are:

1. Market structures are *assumed* by the researcher rather than solved for. Specifying a market structure is the same as specifying the *rules of the game* before predictions (e.g., equilibrium outcomes) are sought.

2. Cournot and Bertrand market structures select Nash equilibrium outcomes, where under the Cournot game firms' actions are the quantity produced, and in the Bertrand game firms' actions are prices.

3. Free entry versus a fixed number of firms.

Self-enforcing collusion (section 6.5) is suited for more advanced students. Instructors may want to go over repeated games (section 2.3) before covering self-enforcing collusion, however, this topic is written in a way that enables the instructor to teach self-enforcing collusion even without formally teaching repeated games. Instructors who choose to teach this topic may want to emphasize that a trigger strategy is a function of the entire history of what *each* player has played in earlier periods, including the player's own earlier actions, and therefore provides self-discipline which restricts the gains from deviation from the cooperative actions.

Finally, subsection 6.6.2 (preferential trade agreements) provides a good example to the law of (no) second best, by showing that a partial removal of trade barriers need not be welfare improving.

Answers to Exercises

1. (a) By Lemma 4.1, both firms produce a finite amount of output only if $p \le c_1$ and $p \le c_2$. Hence, since $c_2 > c_1$, if a competitive equilibrium exists then it must be that $p^e \le c_1$. However, if $p^e < c_1$ then both firms produce zero output, but at these prices demand exceeds zero (since $Q(c_1) = \alpha - c_1 > 0$). Hence, if a competitive equilibrium exists, then $p^e = c_1$. Therefore, $q_1^e = Q^e = \alpha - c_1$ and $q_2^e = 0$.

   (b) Each firm i takes the output of its opponent, q_j, as given and solves

$$\max_{q_i} \pi_i = (\alpha - q_i - q_j)q_i - c_i q_i, \quad i, j = 1, 2, \; i \ne j,$$

   yielding first order conditions given by

$$0 = \frac{\partial \pi_1}{\partial q_1} = \alpha - 2q_1 - q_2 - c_1, \quad \text{and} \quad 0 = \frac{\partial \pi_2}{\partial q_2} = \alpha - q_1 - 2q_2 - c_2.$$

Solving the two equations for the two variables, q_1 and q_2 yields

$$q_1^c = \frac{\alpha - 2c_1 + c_2}{3}, \; q_2 = \frac{\alpha - 2c_2 + c_1}{3}, \; \text{and } Q = \frac{2\alpha - c_1 - c_2}{3}.$$

Then, $p^c = \alpha - q_1^c - q_2^c = (\alpha + c_1 + c_2)/3$.

(c) From the first order conditions of the previous subquestion we can immediately solve for firm 2's best-best response function. Hence,

$$R_2(q_1) = \frac{\alpha - c_2}{2} - \frac{q_1}{2}.$$

In a sequential-moves equilibrium, firm 1 (the leader) takes firm 2's best-response function as given and chooses q_1 to

$$\max_{q_1} \pi_1 = \left[\alpha - q_1 - \left(\frac{\alpha - c_2}{2} - \frac{q_1}{2}\right)\right] q_1 - c_1 q_1,$$

yielding the first order condition

$$0 = \frac{\partial \pi_1}{\partial q_1} = \alpha - 2q_1 - \frac{\alpha - c_2}{2} + q_1 - c_1.$$

Therefore,

$$q_1^s = \frac{\alpha + c_2 - 2c_1}{2}, \; q_2^s = \frac{\alpha - 3c_2 + 2c_1}{4}, \; \text{and } Q^s = \frac{3\alpha - c_2 - 2c_1}{4}.$$

Hence, $p^s = (\alpha + c_2 + 2c_1)/4$.

(d) When firm 2 is the leader, by symmetry (replacing c_1 by c_2 and vice versa in the previous subquestion) we can conclude that

$$q_2^s = \frac{\alpha + c_1 - 2c_2}{2}, q_1^s = \frac{\alpha - 3c_1 + 2c_2}{4}, \; \text{and } Q^s = \frac{3\alpha - c_1 - 2c_2}{4}.$$

Hence, $p^s = (\alpha + c_1 + 2c_2)/4$.

Comparing the two sequential-moves equilibria we see that (i) aggregate output decreases when the less efficient firm (firm 2) moves before firm 1; (ii) price is higher when firm 2 moves first; (iii) the market share of the less efficient firm increases when it moves before firm 1; (iv) the production of the more-efficient firm increases when it moves first; and (v) the market share of the more-efficient firm increases when it moves first.

(e) In a Bertrand equilibrium, the more efficient firm (firm 1) completely undercuts the price set by firm 2. Assuming that money is continuous, firm 1 sets its price to equal the unit cost of firm 2, thereby ensuring that firm 2 will not find it profitable to produce any amount. Hence, we can approximate the Bertrand equilibrium by $p_1^b = c_2 - \epsilon$ and $p_2^b = c_2$. Thus, $q_1^b = Q^b = \alpha - c_2 + \epsilon$ and $q_2^b = 0$. Notice that when the unit costs are not equal (the present case), the Bertrand equilibrium price exceeds the competitive equilibrium price. Also, instructors should point out to the students that this is not really a Nash-Bertrand equilibrium since the profit of firm 1 increases as ϵ decreases.

2. (a) We first focus on the problem faced by firm 1. In a Cournot market structure, firm 1 takes $q_2, q_3, \dots, q_N$ as given and chooses q_1 that solves

$$\max_{q_1} \pi_1 = (100 - q_1 - q_2 - \cdots - q_N)q_1 - F - (q_1)^2.$$

The first order condition is given by

$$0 = \frac{\partial \pi_1}{\partial q_1} = 100 - 2q_1 - q_2 - \cdots - q_N - 2q_1.$$

Therefore, the best-response function of firm 1 is given by

$$q_1 = R_1(q_2, \dots, q_N) = \frac{100 - \sum_{j=2}^{N} q_j}{4}.$$

Given that all firms have identical technologies (identical cost functions), we can attempt to search for a Cournot equilibrium where all firms produce the same output levels. That is, $q^c \equiv q_1^c = q_2^c = \cdots = q_N^c$. In this case,

$$q^c = 25 - \frac{(N-1)q^c}{4}, \quad \text{or } q = \frac{100}{N+3}, \quad \text{hence, } Q = \frac{100N}{N+3}.$$

Therefore,

$$p^c = 100 - \frac{100N}{N+3} = \frac{300}{N+3}, \text{ and } \pi_i = p^c q^c - F - (q^c)^2 = \frac{20,000}{(N+3)^2} - F.$$

(b) Under free entry, firms enter as long as they make above normal profits. Entry stops when a further entry generates a loss to firms. Hence, if we approximate the number of firms, N, by a real number, under free entry all existing firms make zero profit. Thus,

$$0 = \pi_i = \frac{20,000}{(N+3)^2} - F, \text{ or } N = \sqrt{\frac{20,000}{F}} - 3.$$

Thus, the (endogenously determined) number of firms in this industry declines with the fixed cost parameter F.

3. In the third period, firm 3 takes q_1 and q_2 as given and chooses q_3 to maximize its profit. The solution to this problem yields firm 3's best-response function $R_3(q_1, q_2)$ which is given in (6.8). Hence, using (6.8) we have it that

$$R_3(q_1, q_2) = \frac{120 - c}{2} - \frac{q_1 + q_2}{2}.$$

In the second period, firm 2 takes $\bar{q}_1$ and $R_3(\bar{q}_1, q_2)$ as given and chooses q_2 that solves

$$\max_{q_2} \pi_2 = [120 - \bar{q}_1 - q_2 - R_3(\bar{q}_1, q_2)]q_2 - cq_2.$$

The first-order condition is given by $0 = \mathrm{d}\pi_2/\mathrm{d}q_2 = 60 - \bar{q}_1/2 - q_2 - c/2$. Therefore, the best-response function of firm 2 is given by

$$R_2(q_1) = \frac{120 - q_1 - c}{2}.$$

In the first period, firm 1 takes the best-response functions of firms 2 and 3 as given and chooses q_1 that solves

$$\max_{q_1} \pi_1 = \{120 - q_1 - R_2(q_1) - R_3[q_1, R_2(q_1)]\}q_1 - cq_1 = \{30 + \frac{3c}{4} - \frac{q_1}{4}\}q_1 - cq_1.$$

The first-order condition is given by $0 = \mathrm{d}\pi_1/\mathrm{d}q_1 = 30 - c/4 - q_1/2$. Therefore, solving for q_1^s, and then substituting into $R_2(q_1^s)$, and then into $R_3[q_1^s, R_2(q_1^s)]$ yields,

$$q_1^s = \frac{120 - c}{2}, \; q_2^s = \frac{120 - c}{4}, \text{ and } q_3^s = \frac{120 - c}{8}.$$

Hence, $Q^s = q_1^s + q_2^s + q_3^s = 7(120 - c)/8$ and $p^s = (120 + 7c)/8$.

4. (a) Under zero production cost, all Bertrand equilibrium outcomes yield zero profits to both firms. Thus the Bertrand outcome is $p_1^b = 0$ and $p_2^b = 0$.

   (b) We'll construct an equilibrium in trigger strategies where firms' threat point is $p_1^b = 0$ and $p_2^b = 0$, respectively. Formally, let firm i's price strategy, $i = 1, 2$, be given by

   $$p_i(\tau) = \begin{cases} 10 & \text{as long as } p_1(t) = p_2(t) = 10 \text{ for all } t = 1, \ldots \tau - 1 \\ 0 & \text{otherwise.} \end{cases}$$

   Now, if no firm deviates in any period, the present value of the sum of discounted profit to each firm i is $\pi_i = 5N/(1 - \rho)$. If firm 1 deviates in period τ, then $\pi_1 = (10 - \epsilon)N + 0 \approx 10N$ (since effective period $\tau + 1$ all firms revert to $p_i = 0$). Hence, deviation is not profitable to either firm if $5N/(1 - \rho) > 10N$, or $\rho > 1/2$.

   (c) All Bertrand equilibria are of the form $p_1^b = 4 - \epsilon \approx 4$ and $p_2^b = 4$.

   (d) We can reconstruct firms' trigger strategies where we assume that the punishment point is now $p_1^b = 4$ and $p_2^b = 4$. Since now firms have different profit functions, we need to check profitability from deviation by each firm separately.

   Firm 1: In each period τ, if no firm deviates, the sum of discounted profits is $\pi_1 = 5N/(1 - \rho)$. If firm 1 deviates in period τ, then its sum of discounted profit is $\pi_1 = 10N + \rho 4N/(1 - \rho)$. Comparing the two profit levels shows that deviation is not profitable for firm 1 if $\rho > 5/6$.

   Firm 2: If no firm deviates, the sum of discounted profit is $\pi_2 = (N/2)(10 - 4)/(1 - \rho)$. If it deviates, then the sum of discounted profit is $\pi_2 = (10 - 4)N + 0$. Comparing the two profit levels reveals that deviation is not profitable if $\rho > 1/2$.

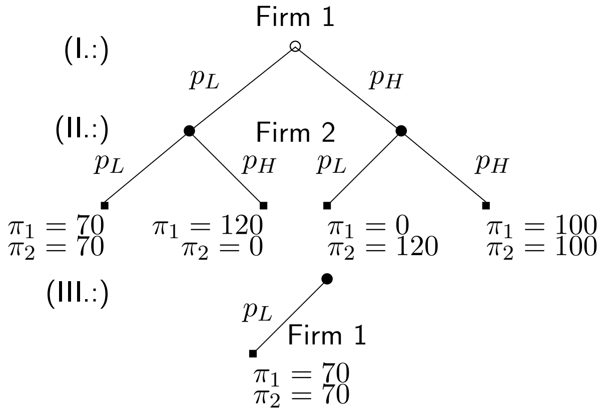
   (e) Collusion is less likely to be sustained when one firm has a cost advantage over the other because deviation by firm 1 (low cost) will leave it with a positive stream of profit compared with a zero profit generated by deviation when the two firms have an identical cost structure.

5. (a) Let p_k^A denote the (tariff inclusive) price consumers in country A pay for a unit of import from country k, $k = B, C$. Therefore, under a uniform specific tariff of \$10 per unit of import, $p_B^A = 70$ and $p_C^A = 50$. Clearly consumers buy from country C, and the government earns a revenue of \$10 per unit. A FTA with country B reduces p_B^A to $p_B^A = 60 > 50 = p_C^A$. Hence, A's consumers still purchase from country C implying that this FTA is ineffective and therefore does not alter A's welfare.

(b) Now, a FTA with country B reduces p_B^A to $p_B^A = 60 < 60.01 = p_C^A$. Hence, A's consumers switch to buying from country B and no tariff revenues are collected. Note that in this case the consumer price falls from $p_C^A = 60.01$ to $p_B^A = 60$ (a reduction of 1 cent per unit of import). Hence, the FTA hardly changes consumer surplus. However, in this case the government faces a large reduction in tariff revenues due to the elimination of a $10 tariff per unit of import. Altogether, with the exception of highly elastic demand, country A loses from the FTA.

6. (a) Clearly p_L is a dominant action for each firm in the static one-shot game. Hence, by Proposition 2.2, (p_L, p_L) is a unique NE for the one-shot game. Therefore, if a SPE exists, it must yield that both firms charge p_L.

We can derive the SPE directly by formulating the extensive form game which is illustrated in Solution-Figure 6.1 (consider the first two stages only).



Solution-Figure 6.1: Sequential price game: Meet the competition clause

From the figure, it is straightforward to conclude that the following prices constitute a SPE:

$$p_2 = \begin{cases} p_L & \text{if } p_1 = p_H \\ p_L & \text{if } p_1 = p_L \end{cases} \quad \text{and} \quad p_1 = p_L.$$

(b) We now add a third stage to the game, where firm 1 commits to reduce its price to firm 2's price whenever firm 2 charges p_L. This third stage is (partly) illustrated at the bottom of Figure 6.1. In this case, the SPE if given by

$$p_2 = \begin{cases} p_H & \text{if } p_1 = p_H \\ p_L & \text{if } p_1 = p_L \end{cases} \quad \text{and} \quad p_1 = p_H,$$

implying that the firms charge the industry's profit maximizing price and earn a profit of 100 each.

7. Note that the answer for exercise 3 provides a solution for $N = 3$ firms, which intuitively can be generalized to the $N > 3$ case. In addition, in what follows, I provide a formal solution for this problem.

(a) Let q_i^s denote the sequential-moves equilibrium output level produced by firm i, $i = 1, \dots, N$. Define $Q^s \equiv \sum_{i=1}^{N} q_i^s$ and $Q_{-N}^s = \sum_{i=1}^{N-1} q_i^s$, for $i = 1, \dots, N$ where $Q_{-1}^s \equiv 0$ for $i = 1$.

Since there is no fixed cost, there is no limit on the number of firms that will enter if free entry is allowed. We therefore look at how the Nth firm decides on how much to produce. Firm N solves

$$\max_{q_N} \pi_N = (a - Q_{-N}^s - q_N - c)q_N,$$

yielding $q_N^s = (a - Q_{-N}^s - c)/2$.

Before we proceed to analyzing firm $N-1$, let us define $a_N \equiv a - Q_{-N}^s$. Hence, it can be verified that

$$a_{N-1} = a_N + q_{N-1}^s \qquad \text{for all} N.$$

Firm $N-1$ knows what firm N will be producing and solves

$$\begin{aligned} \max_{q_{N-1}} \pi_{N-1} &= \left[(a - Q_{-(N-1)}^s - q_{N-1}^s) - \frac{a_N - c}{2} - c \right] q_{N-1} \\ &= \frac{[a_N - c]q_{N-1}}{2} = \frac{[a_{N-1} - q_{N-1} - c]q_{N-1}}{2}. \end{aligned}$$

Hence, $q_{N-1}^s = (a_{N-1} - c)/2$.

Also, since $a_N = a_{N-1} - q_{N-1}^s$ and $a_N = a_{N-1} - q_{N-1}^s = (a_{N-1} + c)/2$,

$$q_N^s = (a_N - c)/2 = (a_{N-1} - c)/4.$$

We now analyze firm $N-2$. Recall that $a_{N-2} - q_{N-2}^s = a_{N-1}$. Also, recall that $q_N^s + q_{N-1}^s = (1/2 + 1/4)(a_{N-1} - c)$. Therefore, firm 2 solves

$$\begin{aligned} \max_{q_{N-2}} \pi_{N-2} &= \left[(a_{N-2} - q_{N-2}) - \frac{3(a_{N-2} - q_{N-2} - c)}{4} - c \right] q_{N-2} \\ &= \frac{[a_{N-2} - q_{N-2} - c]q_{N-2}}{4}. \end{aligned}$$

The solution is $q_{N-2}^s = (a_{N-2} - c)/2$. Hence, $q_{N-1}^s = (a_{N-2} - c)/4$ and $q_N^s = (a_{N-2} - c)/8$. Therefore,

$$q_N^s = \frac{a_N - c}{2}, \qquad \text{for all } N.$$

In particular, for $N = 1$, $a_1 = a - Q_{-1}^s = a - 0 = a$. Thus, $q_1^s = (a - c)/2$.

Finally, since

$$q_N^s = \left(\frac{1}{2}\right)^k q_{N-k}^s, \qquad \text{for all } 0 < k < N,$$

we have it that

$$q_N^s = \left(\frac{1}{2}\right)^{N-1} q_1^s = \frac{1}{2^{N-1}} \frac{a-c}{2} = \frac{a-c}{2^N}.$$

Altogether, the N-firm sequential moves equilibrium output levels are

$$q_i^s = \frac{a-c}{2^i}, \qquad \text{for all } i = 1, \dots, N.$$

(b) Aggregate output is given by

$$Q^s = \sum_{i=1} N q_i^s = (a-c)\left(\frac{1}{2} + \frac{1}{4} + \cdots + \frac{1}{2^N}\right).$$

Using the same arguments as in the proof of Lemma 9.2 (see mathematical appendix, section 9.9), we can show that[1]

$$Q^s = (a-c)\left(1 - \frac{1}{2^N}\right).$$

(c) As the number of firms increases, $Q^s \to a - c$ (competitive output level). Hence, $p^s \to c$ (the competitive price level).

[1]The first printing of the first edition contains a typo in the specification of Q^s in the question.

Chapter 7

Markets for Differentiated Products

The first section extends the market structures defined in Chapter 6 to markets with differentiated products. Using the two-brand environment enables us to analyze the difference in market outcomes generated by the Cournot and the Bertrand market structures, and to define and contrast the concepts of strategically substitutes and strategically complements best-response functions.

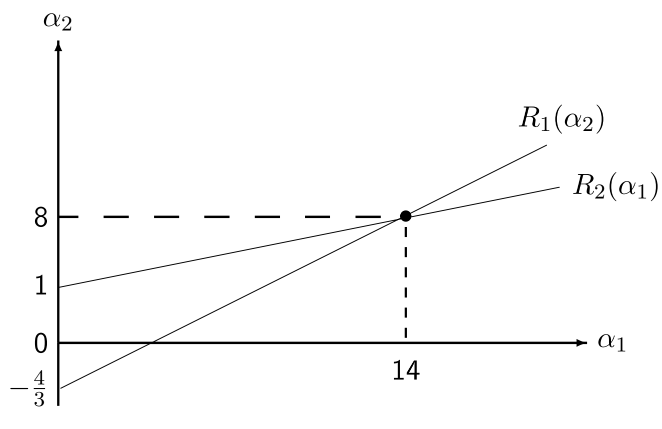
Section 7.2 (somewhat more advanced) introduces the student to the endogenous determination of the variety of brands, and its application to the theory of international trade.

Location models (section 7.3), in particular the Hotelling model (subsection 7.3.1), are important for a further study of industrial organization. The linear city model is used several times in the manuscript, for example, to distinguish between vertically and horizontally differentiated products (see section 12.2). Instructors should cover at least the Hotelling linear city model which students always find very intuitive and therefore very appealing. The existence proof (Appendix, section 7.5) can be skipped. The instructor can demonstrate the parameter restrictions needed to ensure existence by imposing symmetric locations ($a = b$) into the conditions of Proposition 7.6.

Answers to Exercises

1. (a) The best-response functions are drawn in Figure 7.1 and are derived as follows:

$$\max_{\alpha_1} \pi_1 \quad \Rightarrow \quad R_1(\alpha_2) = 2 + \frac{3}{2}\alpha_2, \quad \text{or,} \quad \alpha_2 = -\frac{4}{3} + \frac{2}{3}R_1(\alpha_2);$$
$$\max_{\alpha_2} \pi_2 \quad \Rightarrow \quad R_2(\alpha_1) = 1 + \frac{1}{2}\alpha_1.$$

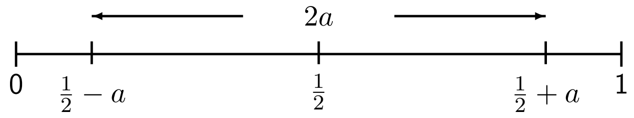


Solution-Figure 7.1: Advertising best-response functions

(b) The two best-response functions are upward sloping, hence strategically complements.

(c) Solving for a NE yields $\alpha_1^N = 14$ and $\alpha_2^N = 8$. The firms' profit levels in a NE are: $\pi_1 = 14^2$ and $\pi_2 = 8^2$.

2. (a) Given a low reservation price (a low B), we'll attempt to find a monopoly equilibrium where not all the market is served. Figure 7.2 illustrates the location of the two indifferent consumers, one on each side of the firm. The indifferent consumer (on



Solution-Figure 7.2: Single-firm location model

each side) is determined by the reservation utility, that is, $B - a - p = 0$, or, $a = B - p$. Hence, the monopoly chooses p that solves

$$\max_p \pi = p2a = 2p(B - p).$$

The solution is given by

$$p^M = \frac{B}{2}, \quad a = \frac{B}{2}, \quad \text{and} \quad \pi^M = 2pa = \frac{B^2}{2}.$$

Now, it is easy to verify that $0 < B < 1$ implies that $0 < a < 1/2$, hence, not all the market is served.

(b) When substituting $B > 1$ into the solution for 'a' found in the previous subquestion, we get that the indifferent consumers lie "outside" the city. This implies that for $B > 1$, the monopoly can increase the price and still having the entire street purchase the product. Thus, the monopoly will pick the highest possible price subject to having the consumers living at the edges of town purchase the product. Formally, set p to satisfy $B - 1/2 - p = 0$. Hence,

$$p^M = B - \frac{1}{2}, \quad a = \frac{1}{2}, \quad \text{and} \quad \pi^M = B - \frac{1}{2}.$$

3. (a) The indifferent consumer, denoted by $\hat{x}$, must satisfy

$$\hat{x} \times 1 + p_1 = (1 - \hat{x}) \times R + p_2.$$

Hence,

$$\hat{x} = \frac{R + p_2 - p_1}{1 + R}.$$

(b) Substituting $p_1 = p_2$ yields $\hat{x} = R/(1 + R)$. Hence,

$$\lim_{R\to\infty} \hat{x} = \lim_{R\to\infty} \frac{R}{1 + R} = 1.$$

Therefore, all the consumers will eat in restaurant 1 (i.e., $\hat{x} = 1$) only if the transportation cost for traveling to the east is infinite. Otherwise, some consumers will always eat at restaurant 2.

4. From (7.24), we can define the location of the indifferent consumer, $\hat{x}$, by the implicit function:
$$F(p_A, p_B) \equiv p_A - p_B + \tau(\hat{x} - a)^2 - \tau(\hat{x} - L + b)^2 = 0.$$
Firm A chooses p_A that solves $\max_{p_A} \pi_A = p_A\hat{x}$ subject to $\hat{x}$ satisfying $F(p_A, p_B) = 0$. The first-order condition is given by
$$0 = \frac{\mathrm{d}\pi_A}{\mathrm{d}p_A} = \hat{x} + p_A\frac{\partial \hat{x}}{\partial p_A} = \hat{x} + p_A\left(-\frac{\frac{\partial F}{\partial p_A}}{\frac{\partial F}{\partial \hat{x}}}\right) = \hat{x} - \frac{p_A}{2\tau(L-a-b)}.$$
Similarly, firm B chooses p_B that solves $\max_{p_B} \pi_B = p_B(L - \hat{x})$, yielding a first-order condition given by
$$0 = \frac{\mathrm{d}\pi_B}{\mathrm{d}p_B} = L - \hat{x} - p_B\frac{\partial \hat{x}}{\partial p_B} = \hat{x} + p_B\left(\frac{\frac{\partial F}{\partial p_B}}{\frac{\partial F}{\partial \hat{x}}}\right) = L - \hat{x} - \frac{p_B}{2\tau(L-a-b)}.$$
   (a) Under $a = b$, we search for a symmetric solution. Substituting $a = b$ and $\hat{x} = L/2$ into firm A's first-order condition (or B's first-order condition) yields that
$$p_A = p_B = \tau L(L - 2a).$$
   Note that $p_A = p_B \to 0$ when $a \to 1/2$, reflecting the fact that prices drop to zero when the two brands become homogeneous.

   (b) We now investigate the total effect of varying the location of firm A (varying a) on the profit of firm A.[1]
   Using the two first-order conditions, we have it that $p_A = 2\tau(L - a - b)\hat{x}$ and $p_B = 2\tau(L - a - b)(L - \hat{x})$. Substituting into the function $F(,) = 0$ yields
$$F(,) = 2\tau(L-a-b)(2\hat{x} - L) + \tau(\hat{x} - a)^2 - \tau(\hat{x} - L + b)^2 = 0.$$
   Using the implicit function theorem, we have it that
$$\frac{\partial \hat{x}}{\partial a} = -\frac{-(2\hat{x} - L) - 2(\hat{x} - a)}{2(L-a-b) + 2(\hat{x} - a) - 2(\hat{x} - L + b)} =\Big|_{\substack{x=1/2\\ a=b}} \frac{1}{4}.$$
   Now, using A's first-order condition, we can express A's profit by $\pi_A = p_A\hat{x} = 2\tau(L-a-b)(\hat{x})^2$. Therefore,
$$\frac{\mathrm{d}\pi_A}{\mathrm{d}a} < 0 \quad \text{whenever} \quad -(\hat{x})^2 + (L-a-b)2\hat{x}\frac{1}{4} < 0,$$
   which always hold since $L - 2a < L$. Hence, firm A increases its profit by moving toward point 0. Similarly, firm B increases its profit by moving toward point L.

[1] This problem may be too difficult for most undergraduate students since it involves deviating from the symmetric solution.

Chapter 8

Concentration, Mergers, and Entry Barriers

This chapter contains four major topics: How to measure concentration; merger and merger guidelines; entry barriers; and entry deterrence. I urge the instructor not to skip the discussion of concentration measures (section 8.1) and the appendix on merger regulations and guidelines (section 8.6).

Regarding all other topics, the instructor will find a wide variety of models to choose from. Most of the topics are logical and require minimum calculations and very little calculus (if any).[1] Finally, note that you can combine the topic of double monopoly markup, analyzed under the topic of dealership (subsection 14.3.1), with the analysis of vertical integration given in subsection 8.2.2.

Answers to Exercises

1. (a) $I_4 = 20 + 20 + 20 + 10 = 70$.
   (b) $I_{HH} = 4 \times 10^2 + 3 \times 20^2 = 1,600$.
   (c) i. After the merger of firms 1 and 2, $I_{HH} = 20^2 + 10^2 + 10^2 + 3 \times 20^2 = 1,800$.
       ii. $\Delta I_{HH} = 1,800 - 1,600 = 200$.
       iii. The merger may be challenged by the FTC or the Justice Department since the postmerger I_{HH} exceeds $1,000$ and $\Delta I_{HH} = 200 > 100$.

2. (a) The solution for a Cournot equilibrium with N firms is given in (6.9) and (6.10). Substituting $c = 0$ (zero production cost) and $N = 3$ yields $q_i^c = 25$ and $\pi_i^c = 625$, $i = 1, 2, 3$.
   (b) Substituting $N = 2$ into (6.10) yields $\pi_1 = \pi_4 = 1111.111\ldots$.
   (c) Since firms 2 and 3 split the profit from the merged firm, $\pi_2 = \pi_3 = \pi_4/2 = 555 < 625$. Hence, firms 2 and 3 lose from this merger whereas firm 1 gains.
   (d) A merger into a monopoly cannot reduce aggregate profit, simply because a monopoly can always mimic the same production pattern of any oligopoly. To see this, substituting $N = 1$ into (6.10) yields a monopoly profit of $\pi^M = 2500$. Hence, $\pi_i = 2500/3 = 833\frac{1}{3} > 625 > 555$. Thus, the profit per firm is the highest under the monopoly market structure (cartel).
   (e) The difference between the two mergers is as follows: When two out three firms merge, competition is reduced to a duopoly; however, the extra aggregate industry profit generated from the change in the number of firms is equally divided between

[1] Subsection 8.4.2 can be presented using diagrams only. However, the first printing of the first edition lacks a justification why firm 1's (high and low) best-response functions do not shift with a change in $\hat{k}$. In the second printing I intend to add an appendix at the end of this chapter which derives both best-response functions. Also, note that the equilibrium first-period capital level in Figure 8.7 is $\hat{k}_3$ and not $\hat{k}_2$ as wrongly stated in the first printing.

the two unmerged groups of firms. Thus, this share of the change in aggregate industry profit is lower than the profit each firm makes under the triopoly.

In contrast, merging into a monopoly can only increase profit since a monopoly can always produce like a duopoly, and under monopoly, all industry profit is equally divided between the three firms.

3. (a) We look for a Nash equilibrium in prices.[2] The X-seller takes p_Y as given and solves

$$\max_{p_X} \pi_X = p_X Q = p_X(\alpha - p_X - 2p_Y), \quad \text{yielding} \quad p_X = \frac{\alpha}{2} - p_Y.$$

The Y-seller takes p_X as given and solves

$$\max_{p_Y} \pi_Y = p_Y 2Q = 2p_Y(\alpha - p_X - 2p_Y), \quad \text{yielding} \quad p_Y = \frac{\alpha - p_X}{4}.$$

Hence,

$$p_X^N = \frac{\alpha}{3}, \; p_Y^N = \frac{\alpha}{6}, \; p_S^N = \frac{2\alpha}{3}, \; Q^N = \frac{\alpha}{3}, \; \pi_X^N = \pi_Y^N = \frac{\alpha^2}{9}.$$

(b) We define a system as one unit of X bundled with two units of Y. Then, the merged firm chooses a system price p_S that solves

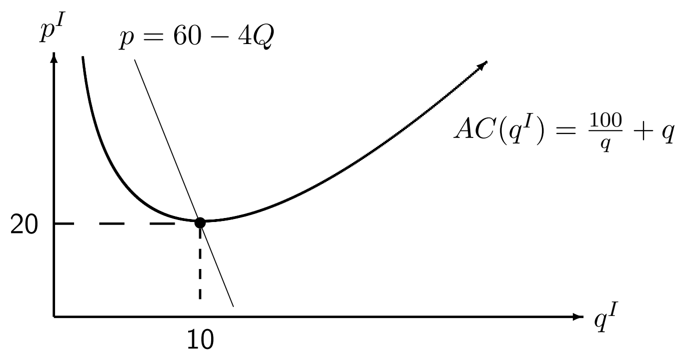
$$\max_{p_S} \pi_S = p_S(\alpha - p_S) \quad \text{yielding} \quad p_S = Q_S = \frac{\alpha}{2}, \text{ and } \pi_S = \frac{\alpha^2}{4}.$$

(c) To make a welfare judgment on the gains from this merger it is sufficient to compare prices and profit levels. Before the merger, the price of one system is $p_S^0 = p_X + 2p_Y = 2\alpha/3 > \alpha/2 = p_S^1$ which is the price after the merger. Also, before the merger, aggregate profit is $\pi_X + \pi_Y = 2\alpha^2/9 < \alpha^2/4 = \pi_S^1$ which is the profit after the merger takes place. Since the system price falls and industry profit increases, the merger is welfare improving.

4. In view of Definition 8.1, Figure 8.1 shows that the pair $(p^I, q^I) = (20, 10)$ constitutes a contestable-market equilibrium.

Formally, the (incumbent's) industry configuration $(p^I, q^I) = (20, 10)$ is feasible since demand equals supply and the incumbent makes a nonnegative profit. It is also sustainable since no potential entrant can undercut the incumbent's price while making a positive profit.

[2] A typo in the first printing of the first edition: the question should state that $Q = x = y/2$.



Solution-Figure 8.1: Contestable-markets equilibrium

Chapter 9

Research and Development

Most of the topics in this chapter are not technically demanding (and less tedious in terms of technical derivations). The classifications of process innovation (section 9.1) uses only demand and supply diagrams. Innovation race (section 9.2) relies on simple discrete probability calculations. Cooperation in R&D (section 9.3) relies on a Cournot market structure in the second stage and a Nash equilibrium in R&D in the first stage. The calculation of the optimal patent life (section 9.4) does not reach a closed-form solution. However Figure 9.3 enables the instructor to explain the cost and benefit of the patent system without performing any calculations. Then, the instructor can simply reproduce the government-innovation two stage game and demonstrate how to build the social welfare function for calculating the optimal patent life. Licensing of innovation (section 9.5) uses supply and demand analysis. The international analysis (section 9.6) is divided into a simple matrix game of product innovation and a two stage subsidy Cournot game of process innovation. Finally, the appendix sections discuss patent law and R&D joint ventures (sections 9.7 and 9.8) and can be assigned as homework reading.

Answers to Exercises

1. Using Definition 9.1, innovation is **major** if the monopoly price is below the initial unit cost; that is if $p^M < c_0$. Innovation is minor if $p^M > c_0$. To find p^M, the monopoly solves $MR(Q^M) = a - 2Q^M = c_1 = 2c_0 - a$. Therefore, the monopoly's profit maximizing output is $Q^M = a - c_0$. Hence, $p^M = c_0$. Therefore, according to Definition 9.1 the present innovation is neither major nor minor.

2. (a) The expected profit of a firm engaged in R&D, given that the other two firms are also engaged in R&D, is the prize times the sum of the probability that the firm discovers while the other two do not, plus twice the probability that it discovers while one competing firm discovers and one does not, plus the probability that all the three discover simultaneously, minus innovation cost. Formally, for each firm i, $i = 1, 2, 3$,

$$\mathsf{E}\pi_i = \frac{1}{2}\frac{1}{2}\frac{1}{2}V + \frac{1}{2}\frac{1}{2}\frac{1}{2}\frac{V}{2}(\times 2) + \frac{1}{2}\frac{1}{2}\frac{1}{2}\frac{V}{3} - I = \frac{7V}{24} - I.$$

Thus, all the three firms find it profitable to engage in R&D if $V > 24I/7 = 24/7$ (since $I = 1$).

(b) The now single firm can operate zero, one, or two labs.

Operating a single lab: In this case, $\mathsf{E}\pi|_{\text{1 lab}} = V/2 - I$.

Operating two labs: In this case, the probability of discovery is one minus the probability that none of the labs discovers. Thus,

$$\mathsf{E}\pi|_{\text{2 labs}} = \left(1 - \frac{1}{2}\frac{1}{2}\right) V - 2I = \frac{3V}{4} - 2I.$$

Now, $\mathsf{E}\pi|_{\text{2 labs}} > \mathsf{E}\pi|_{\text{1 lab}}$ if $V > 4I$, which constitutes a sufficient condition for having the firm choosing to operate two labs.

3. (a) The probability that a firm does not discover is $(1-\alpha)$. Hence, the probability that none of the n firms discovers at a particular date is $(1-\alpha)^n$.

   (b) Clearly, one minus the probability that none discovers. Hence, $1-(1-\alpha)^n$.

   (c) Similar to the derivation given in (9.5),

$$\begin{aligned}\mathsf{E}T(n) &= [1-(1-\alpha)^n]\times 1 + (1-\alpha)^n\,[1-(1-\alpha)^n]\times 2\\ &+ (1-\alpha)^2 n\,[1-(1-\alpha)^n]\times 3 + (1-\alpha)^3 n\,[1-(1-\alpha)^n]\times 4+\cdots\end{aligned}$$

   Therefore,

$$\mathsf{E}T(n) = [1-(1-\alpha)^n]\sum_{t=1}^{\infty}[(1-\alpha)^n]^{t-1}\times t \overset{\text{(Lem 9.1)}}{=} \frac{1-(1-\alpha)^n}{[1-(1-\alpha)^n]^2} = \frac{1}{\alpha(2-\alpha)}.$$

4. (a) If the EC provides Airbus with a subsidy of 10 (contingent on developing the megacarrier, then Airbus' dominant action is to develop.

   (b) If the U.S. government provides Boeing with a subsidy of 10 (contingent on developing the megacarrier, then Boeing's dominant action is to develop. Note that this U.S. subsidy is independent whether the EC provides Airbus with a subsidy of 10 or 15.

   (c) None! After the EC subsidy to Airbus is implemented, Airbus' dominant action is to develop (hence, independent of the U.S. subsidy to Boeing).

   (d) Like any other welfare argument in economics, there are pros and cons to having both governments subsidizing the development of the new megacarrier. Arguments against the subsidies include (i) duplication of R&D which require twice the amount of resources needed to develop and test the plane, (ii) the government may not possess the means to determine whether the benefits from the development to their own economy dominate the cost, (i.e., economists generally believe that the private sector can collect more accurate information), and (iii) income distribution effects; that is, some economists claim that the development of the Concord by some EC countries constitute a net transfer from poor to rich people, since most poor and middle-class people do not get to fly in the Concord.

   Arguments for the subsidy include the introduction of more competition between aircraft manufacturers that would reduce aircraft prices, boost the airline industry, and may therefore increase the amount of travel.

Chapter 10

The Economics of Compatibility and Standards

The introduction to this chapter provides some basic terminology and a simple "Battle of the Sexes" game of product standardization, which can be taught to the less technically proficient students. The behavior of a monopoly telephone company under network externalities (Subsection 10.1.1) uses very simple calculus (basically one first-order condition). The remainder of section 10.1 is devoted to a simple discrete (noncalculus) standardization-variety tradeoff model. The remaining two sections, the supporting services approach (section 10.2) and the components approach (section 10.3) are somewhat more difficult despite the fact that no calculus is used. In particular, whereas the components approach model is very simple, some students may get confused because of the existence of multiple equilibria under incompatibility, in which case, the instructor may want to construct only one equilibrium and compare it to the equilibrium under compatible systems.

The important points to emphasize are:

- A network externality is a particular type of a consumption externality, where a consumer's utility is affected by the *number* of consumers purchasing the same brand. Thus, network externalities constitute an *assumption* about consumer preferences.
- The supporting services and the components approaches do not assume any externality. The "network effects," where the utility of a consumer is affected by the brand choices of other consumers is an equilibrium result (rather than an assumption).[1] Thus, the network effects are generated by having hardware and software (or other types of hardware) treated as perfect complements.

Answers to Exercises

Unfortunately, the two exercises cover only the supporting services and the components approaches. In future printings/editions, I intend to add two exercises covering the two topics analyzed in the network externalities section.

1. (a) Firm A gains control over the entire market when $\hat{\delta} = 1$. In this case, by (10.10), p_B has to be sufficiently high to satisfy

$$1 = \hat{\delta} = \frac{Y - p_A}{2Y - p_A - p_B}, \quad \text{or} \quad p_B = Y.$$

   (b) i. Let $\bar{\delta}$ denote the market share for firm A after income has doubled from Y to $2Y$. Then, by (10.10) we have that

$$\bar{\delta} = \frac{2Y - p_A - p_B}{4Y - p_A - p_B} > \hat{\delta} = \frac{Y - p_A - p_B}{2Y - p_A - p_B}$$

[1] Some authors use the term *indirect network externalities* to describe the behavior generated by the supporting services and the components approaches. I feel that this term was poorly chosen since externalities are either assumed or not assumed at all, and therefore the term *indirect* cannot be used to describe models that do not explicitly assume any externality.

if $p_A > p_B$. This result can also be demonstrated by differentiating $\hat{\delta}$ with respect to Y, yielding

$$\frac{\mathrm{d}\hat{\delta}}{\mathrm{d}Y} = \frac{p_A - p_B}{(2Y - p_A - p_B)^2} > 0 \quad \text{if} \quad p_A > p_B.$$

Thus, the firm with the lower market share increases its market share when consumer income rises.

ii. Since $\hat{\delta}$ increases, by (10.9) N_A/N_B must increase.

2. (a) Potential equilibria can be classified into two types: (i) Firm A sells to consumers AA, and AB; and firm B sells to BB and BA (alternatively, Firm A sells to consumers AA, and BA; and firm B sells to BB and AB). (ii) Firm A sells to AA, AB, and BA; whereas firm B sells to BB only (alternatively, firm A sells to AA only, whereas firm B sells to BB, AB, and BA).

In what follows we will demonstrate, by a way of contradiction, that both types of equilibria do not exists.

i. Under this type, $p^I_A = p^I_B$ since consumers AB and BA are indifferent between systems AA and BB as long as the prices are equal. However, this prices cannot support a Bertrand-Nash equilibrium since firm A can increase its profit by undercutting firm B by charging $p_A = p^I_B - \epsilon$ which will cause consumer BA to switch to system AA (this is the usual Bertrand undercutting argument).

ii. Under this type, we must have that $p^I_B = p^I_A + 2\lambda$ (otherwise firm B is not maximizing its profit). However, since consumers' reservation utility is zero, see equation (10.12), equilibrium prices must satisfy $p^I_B \le 2\lambda$, hence, $p^I_A = 0$ and $\pi^I_A = 0$. However, firm A can always increase its profit by raising its price by ϵ thereby making strictly positive profit.

(b) Following the proof of Proposition 10.14, it can be established that there exists an equilibrium in which each consumer buys his or her ideal system. In this equilibrium,

$$p^x_A = p^y_A = p^x_B = p^y_B = \lambda, \quad \text{and} \quad \pi^c_A = \pi^c_B = 4\lambda.$$

Chapter 11

Advertising

From a technical point of view, the analyses given in this chapter are pretty much straight forward, perhaps, with the exception of the calculation of the socially optimal level of persuasive advertising given in equation 11.13, where an integral is computed. The appendix describing advertising regulations can be assigned as a home reading.

Answers to Exercises

1. (a) Since

$$\epsilon_A = \frac{\%\Delta Q}{\%\Delta A} = 0.05, \quad \text{and} \quad \epsilon_p = \frac{\%\Delta Q}{\%\Delta p} = -0.2,$$

then by the Dorfman-Steiner condition (Proposition 11.1), we have that the profit maximizing ratio of advertising to revenue is

$$\frac{A^M}{pQ^M} = \frac{\epsilon_A}{-\epsilon_p} = \frac{1}{4}.$$

Hence, $A^M = \$2.5$ mil..

(b) Now, $\epsilon_p = -0.5$. Therefore $A^M = 10 \times 0.05/0.5 = \$1$ mil..

(c) When the demand becomes more price elastic, the monopoly reduces the ratio of advertising expenditure to revenue.

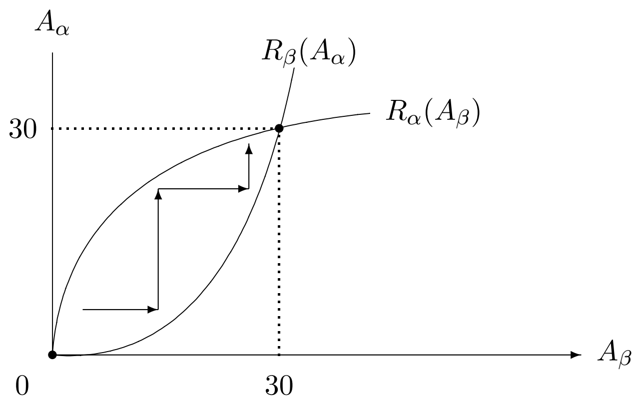
2. (a) $A_\alpha = A_\beta = 1$ implies that $n_\alpha = n_\beta = 3$, hence, $\pi_\alpha = \pi_b = 30 - 1 = 29$. When $A_\alpha = A_\beta = 2$, we have it that $n_\alpha = n_\beta = 3$, hence, $\pi_\alpha = \pi_b = 30 - 2 = 28$. Conclusion: if both firms spend the same amount on advertising, then industry profit is maximized at $A_\alpha = A_\beta \to 0$. That is, advertising in this city serves as a mean by which each firm attracts consumers from the competing firm, but cannot increase industry sales beyond the level determined by zero advertising.

(b) Fortune teller β takes A_α as given and chooses A_β to maximize his or her profit. The first-order condition is given by

$$0 = \frac{\partial \pi_\beta}{\partial A_\beta} = \frac{30 A_\alpha}{(A_\beta)^2} - 1.$$

The second-order condition can be easily verified. Hence, $A_\beta = R_\beta(A_\alpha) = \sqrt{30 A_\alpha}$. By symmetry, $A_\alpha = R_\alpha(A_\beta) = \sqrt{30 A_\beta}$. The two best-response functions are drawn in Solution-Figure 11.1.

(c) Solution-Figure 11.1 illustrates that the two best response functions intersect twice, reflecting the fact that there exist two Nash equilibria for this advertising game. For the purpose of this question, we define $0/0$ (yes, zero divided by zero) as equal to 1.



Solution-Figure 11.1: Fortune tellers' advertising best-response functions

Then, clearly $A_\alpha = A_\beta = 0$ constitutes one NE. To obtain the other NE, substituting $R_\alpha(A_\beta)$ into R_β yields $A_\beta = \sqrt{30\sqrt{30A_\beta}}$. Hence, $A_\alpha^N = A_\beta^N = 30$ which is also illustrated in Solution-Figure 11.1.

(d) In the interior NE, $\pi_i^N(30, 30) = 30 - 30 = 0$, $i = \alpha, \beta$, which is lower than $\pi_i^N(1, 1) = 29$. Thus, the industry's profit is not maximized at the interior NE. However, at the other NE, industry's profit is maximized and each firm earns $\pi_i^N(0, 0) = 30$.

(e) The interior NE $(30, 30)$ is stable. The interior NE $(0, 0)$ is unstable since any unilateral deviation by one of the firm would trigger a sequential chain of responses that would move the advertising equilibrium to $(30, 30)$ (see Solution-Figure 11.1 for an illustration). Thus, in this example, the NE that maximizes industry profit is unstable.

3. In order for (I, P) to constitute a NE outcome, we need to show that unilateral deviation is not profitable for each firm. That is, using Table 11.1,

$$\pi^2(I, P) = N \ \geq \pi^1(I, I) = (1 - \theta)E \quad \text{when } \theta \geq 1 - N/E;$$
$$\pi^1(I, P) = \theta E \ \geq \pi^1(P, P) = N/2 \quad \text{when } \theta \geq N/(2E).$$

Chapter 12

Quality, Durability, and Warranties

This is a long chapter with a wide variety of topics dealing with supply and demand for quality. The analyses are pretty much straight forward and do not require any preparation with the exception of section 12.2 that relies on the knowledge of location models developed in Chapter 7. Other sections use only simple calculations of expected values. The appendix on the legal approach to liability (section 12.8) may be assigned as a home reading.

An Alternative Model for Section 12.6

The model suggested below provides some improvement as it assumes that consumers are *expected* utility maximizers (similar to what is assumed in Section 12.5).[1] Suppose that consumers believe that the monopoly is a high-quality producer with probability α, and a low-quality producer with probability $1-\alpha$, where $0 < \alpha < 1$. Then, without any additional information, $\mathrm{E}U = \alpha H + (1-\alpha)L - p$. Therefore, the maximum price that consumers are willing to pay is $\bar{p} = \alpha H + (1-\alpha)L$. Then a low-quality producer could set the price at $\bar{p}$, and make a profit of

$$(\bar{p} - c_L)\cdot 1 = \alpha H + (1-\alpha)L - c_L = \alpha H + (1-\alpha)L - c_L > 0. \qquad (*)$$

The price and quantity pair in Proposition 12.10 should be modified to

$$p^m = H \quad \text{and} \quad q^m = \frac{\alpha H + (1-\alpha)L - c_L}{H - c_L}.$$

Had the monopolist been a low-quality producer, he or she could have made the same profit by setting $\bar{p}$ and selling to all consumers [as is indicated in (*)] rather than setting $p = p^m$ and $q = q^m$. Therefore, in choosing this price-quantity pair the monopolist could signal his or her quality to the consumers. Finally, since $H > L > c_H > c_L$,

$$(H - c_H)\frac{\alpha H + (1-\alpha)L - c_L}{H - c_L} > \alpha H + (1-\alpha)L - c_H \;= (\bar{p} - c_H)\cdot 1.$$

Hence, the high-quality monopoly would indeed find it profitable to signal its high quality to the consumers.

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[1] I thank Keiichi Koda for proposing this improvement.

Answers to Exercises

1. (a) Using the utility function (12.2) and the zero reservation utility assumption, the consumer who is indifferent between purchasing from A and not purchasing at all must satisfy $U_z(A) = az - p_A = 0$, yielding $z = p_A/a$.

   (b) Follows directly from (12.4).

   (c) For a given p_B, firm A chooses p_A to

$$\max_{p_A} \pi_A = p_A \left[\frac{p_B - p_A}{b - a} - \frac{p_A}{a}\right], \quad \text{yielding} \quad 0 = \frac{p_B - 2p_A}{b - a} - \frac{2p_A}{a}.$$

   For a given p_A, firm B chooses p_B to

$$\max_{p_B} \pi_B = p_B \left[1 - \frac{p_B - p_A}{b - a}\right], \quad \text{yielding} \quad 0 = 1 - \frac{2p_B - p_A}{b - a}.$$

   Second-order conditions are easily obtained. Solving the two first-order conditions yield the equilibrium prices. Substituting into the profit functions yield the equilibrium profit levels.

   (d) We first verify that given $b^e = 1$, the profit maximizing location of firm A is $a^e = 4/7$. Thus, given $b^e = 1$, firm A chooses a^e to

$$\max_{a} \pi_a = \frac{a(1-a)}{(4-a)^2}, \quad \text{yielding} \quad a^e = \frac{4}{7}.$$

   We now verify that given $a^e = 4/7$, firm B would choose to locate on the east edge of the street.[2] For a given a^e firm B chooses b^e to maximize π_B which is given above. We would like to show that for $a^e = 4/7$,

$$0 < \frac{\partial \pi_B(4/7, b)}{\partial b} \approx 49b^2 - 21b + 8 \equiv \phi(b).$$

   To demonstrate this inequality, we need to prove that $\phi(b) > 0$ for all $b \in [0, 1]$. Clearly $\phi'(b) = 98b - 21$, which is negative for $0 \le b < 21/98$ and positive for $b > 21/98$. Hence, the function ϕ hits a minimum at $b = 21/98$. Now, $\phi(21/98) = 5.75 > 0$. Hence $\phi(b) = \partial\pi_B/\partial b > 0$ for all $b \in [0, 1]$ implying that firm B would locate at $b^e = 1$.

2. Let $U_i(k)$ denote the utility of consumer i when he buys the brand with quality k, $k = H, L$ and $i = 1, 2$. We want to show that

$$U_2(L) = L(I_2 - p_L) > H(I_2 - p_H) = U_2(H).$$

   From (12.1), we have it that since consumer 1 buys the low-quality brand, then

$$U_1(L) = L(I_1 - p_L) > H(I_1 - p_H) = U_1(H).$$

   Therefore,

$$(H - L)I_1 < Hp_H - Lp_L.$$

   Now, since $I_2 < I_1$ by assumption, $(H-L)I_2 < Hp_H - Lp_L$, implying that $H(I_2 - p_H) < L(I_2 - p_L)$.

[2] Many students will find this part of the proof to be difficult.

3. Under these prices, we seek to characterize the demand and supply patterns of our agents.[3]

   New buyers (who do not yet own a car): Recalling our assumption that $N^L = U^L = 0$, if a new buyer buys a new car, then his or her expected utility is $V^b = 0.5N^G + 0.5N^L - \bar{p}^N = 0$. In contrast, if a new buyer buys a used car then $V^b = 0.5U^G + 0.5U^L - \bar{p}^U > 0$. Hence, under the assumed prices, new buyers buy used cars.

   Good-used-car sellers: The question simply assumes that good-used-car owners must leave the country and therefore sell their good-used cars.

   Lemon-used-car sellers: If a lemon-used-car seller sells his or her lemon and buys a new car, then $V^{s,L} = 0.5N^G - \bar{p}^N + \bar{p}^U = \bar{p}^U$. In contrast, if she or he buys a used car then $V^{s,L} = 0.5U^G - \bar{p}^U + \bar{p}^U = 0.5U^G$. By assumption $\bar{p}^U < 0.5U^G$, hence, lemon-used-car sellers buy (and sell) used cars.

   Altogether, under the assumed prices, nobody buys a new car.

4. (a) The monopoly's production cost is the initial production cost plus the expected (as most once) replacement cost, which is equal to $c + (1 - \rho)c = (2 - \rho)c$.

   (b) The maximum monopoly price is the expected consumer surplus from purchasing this product with this type of warranty. This surplus equals V if the product does not fail at all and if the product fails only once (in which case it is replaced). The probability of a double failure is $(1 - \rho)^2$. Hence, the probability of no double failure is $1 - (1 - \rho)^2 = \rho(2 - \rho)$. Therefore, $p^M = \rho(2 - \rho)V$.

   (c) If the monopoly provides this warranty, then $\pi^W = \rho(2 - \rho)V - (2 - \rho)c = (2 - \rho)(\rho V - c)$. If the monopoly sells with no warranty, then $\pi^{NW} = \rho V - c$. Hence, $\pi^W > \pi^{NW}$ since it is assumed that $\rho V > c$. Thus, the monopoly will bundle the product with this warranty.

[3] The purpose of this question is demonstrate that used cars are traded when some good-used-car owners are "forced" to sell their car, say, because they leave the country. *Note:* If you are using the first printing of the first edition, change the last sentence in this question to "...the four *types* of agents...."

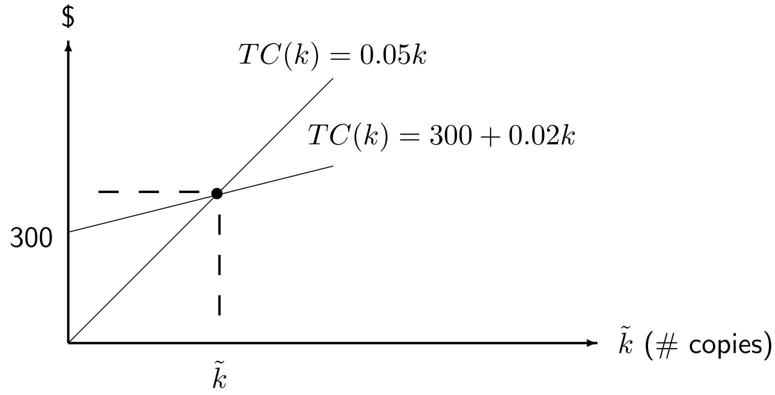
Chapter 13

Pricing Tactics: Two-Part Tariff and Peak-Load Pricing

The analyses given in this relatively short chapter rely mainly on logical arguments and therefore do not require any special technical preparation. Section 13.4 presents an attempt to endogenize the seasons in determining profit maximizing peak-load pricing, and therefore may be given a lower priority in case the instructor is short of time.[1]

Answers to Exercises

1. (a) The two cost functions are drawn in Solution-Figure 13.1.



Solution-Figure 13.1: The cost of photocopying with or without two-part tariff

(b) The number of copies yielding identical cost under the two payment systems is found by equating $0.05\hat{k} = 300 + 0.02\hat{k}$. Therefore, $\hat{k} = 10,000$ copies. Thus, for $k > \hat{k}$, the two-part tariff system is less costly, and for $k < \hat{k}$, the fixed-per-unit price system is less costly.

2. (a) By Proposition 13.4, to find the profit maximizing pricing structure we need (i) to equate the "high-season" marginal revenue to the cost of an aircraft seat plus the per-passenger cost, and (ii) to equate the "low-season" marginal revenue to the per-passenger cost only.

Given that the Summer demand curve lies completely below the Winter demand curve, we can conclude that Winter is the high season. However, since the two demand curves are "relatively close," we need to be careful about the capacity constraint determined by the Winter condition.

In the Winter, capacity is determined by having the monopoly equating

$$MR_W(q_W) = 10 - 2q_W = r + c = 2, \quad \text{and} \quad MR_S(q_S) = 5 - q_S = c = 1.$$

[1] If you are using the first printing of the first edition note that the calculations of consumer surplus in section 13.2 are wrong. Ask for the errata file by e-mail.

The first equality implies that $q_W = 4$, hence, $p_W = 6$. The second equality implies that $q_S = 4$, hence, $p_S = 3$. Altogether, $\pi^I = 6 \times 4 + 3 \times 4 - 2 \times 4 - 1 \times 4 = 24$.

(b) Similar to the previous subquestion, the monopoly equates

$$MR_W(q_W) = 10 - 2q_W = r + c = 4, \quad \text{and} \quad MR_S(q_S) = 5 - q_S = c = 1.$$

The first equality implies that, $q_W = 3$, hence, $p_W = 7$. The second equality implies $q_S = 4 > 3 = q_W$. Hence, due to the capacity constraint, the airline will fly only 3 passengers during the Summer season. Thus, $p_S = 5 - 3/2 = 3.5$. Altogether, $\pi^I = 7 \times 3 + 3.5 \times 3 - 4 \times 3 - 1 \times 3 = 16.5$.

Chapter 14

Marketing Tactics: Bundling, Upgrading, and Dealerships

This chapter introduces several topics related to profit enhancing marketing tools that were not discussed (at least not in full) in earlier chapters.

Answers to Exercises

1. (a) Under pure tying, we need to check two options: (i) Under $p^T = 4$, all three consumers buy the packages, hence, $\pi^T(4) = 3 \times 4 = 12$. (ii) Under $p^T = 10$, only consumer 2 buys a package, hence, $\pi^T(10) = 10$. Clearly, the first option maximizes the profit under pure tying.

   (b) The profit maximizing product and package prices are $p_X^{MT} = p_Y^{MT} = 4$ and $p^{MT} = 8$. Under this price structure, consumer 1 buys one unit of X, consumer 3 buys one unit of Y and consumer 2 buys the package (or one unit of each good). Altogether, $\pi^{MT} = 4 + 4 + 8 = 16$.

   (c) Notice that mixed tying yields the same profit maximizing pricing structure as no tying, since in the present example, the monopoly cannot charge consumer 2 more than the sum of the prices of the two products sold separately. In fact, in the present example, pure tying is profit reducing. Therefore, mixed tying (which yields an identical allocation as no tying) generates a higher profit than pure tying.

2. (a) We need to compare the profit levels under the commitment of not introducing a new edition, in which case the publisher can charge a higher price in the first period by adding the resale value to the price, to the profit level when no commitment is made and the monopoly introduces a new edition.

   By (14.17), the profit level under commitment is nV, and under no commitment is $2n(V - c) - F$. Comparing the two profit levels reveals that commitment not to introduce a new edition is more profitable if $F > nV - 2c$.

   (b) Indeed, this condition in weaker (i.e., implied by) the condition given in (14.17) since it compares in the case of commitment, in which first-period students believe that no new edition will be published, hence the monopoly can raise the book's price by its resale value, to the case where there is no commitment and a new edition is introduced.

   In contrast, the condition given in (14.16) and hence in (14.17) is derived by comparing the monopoly's second period profit only and therefore yields the condition under which a new edition will not be introduced in the second period. In other words, the condition given in (14.16) and (14.17) "does not take into account" that first period price can be raised by the resale value when a commitment is made not to introduce a new edition.

3. (a) When a new edition is not introduced in the second period, the monopoly competes with period 1 students on selling the first edition. First period students will undercut

the monopoly (since first-period students do not have any production cost) and for a given p_2^N will set p_2^U so that

$$V - p_2^N \le \alpha V - p_2^U.$$

Now, in a Bertrand equilibrium, if used-book sellers can undercut the monopolist, then the monopoly price is reduced to cost($p_2^N = c$). Hence, the maximum price user-book sellers can charge is

$$p_2^U = \begin{cases} c - (1-\alpha)V & \text{if } c \ge (1-\alpha)V \\ 0 & \text{otherwise} \end{cases}$$

Notice that if $c < (1-\alpha)V$, or $\alpha < 1 - c/V$, then the monopoly undercuts the used-book sellers (since consumers do not value used books very much). Under this condition, the monopoly would set $p_2^N = (1-\alpha)V$, and therefore earn a profit of $\pi_2^N = n[(1-\alpha)V - c]$.

(b) There are two cases:

High value for used books ($\alpha > 1 - c/V$): A new edition would yield a second-period profit of $\pi_2^N = n(V-c) - F$. When a new edition is not introduced, then $\pi_2 = 0$. Hence, a new edition is introduced if $F \le n(V-c)$, which is the same as condition (14.16) when $\alpha = 1$.

Low value for used book ($\alpha < 1 - c/V$): When α is low, used book sellers are undercut by the monopoly. In this case, if a new edition is not introduced in the second period, then $p_2^N = (1-\alpha)V - c$ (the monopoly undercuts used-book sellers). Hence, $\pi_2^N = n[(1-\alpha)V - c]$. Altogether, a new edition is introduced if $n(V-c) - F \ge n[(1-\alpha)V - c]$, or if $F \le n\alpha N$, which is different from condition (14.16).

4. (a) Under this contract, the dealer chooses Q (sales) to

$$\max_Q \pi^d = (1-\phi)[p(Q)Q - cQ] = (1-\phi)[(a-Q)Q - cQ].$$

The first-order condition yields $Q^d = (a-c)/2$, $p^d = (a+c)/2$, and $\pi^d = (1-\phi)(a-c)^2/4$. Thus, the dealer's sales and price are *independent* of $(1-\phi)$ (share of the profit of the dealer). Under this contract, the industry profit is

$$\pi^M + \pi^d = \phi\frac{(a-c)^2}{4} + (1-\phi)\frac{(a-c)^2}{4} = \frac{(a-c)^2}{4},$$

which equals the industry's profit when the firms vertically integrate into a monopoly given in (14.24).

(b) Under this contract, the dealer chooses Q that solves

$$\max_Q \pi^d = (1-\phi)p(Q)Q - cQ = (1-\phi)(a-Q)Q - cQ.$$

The first-order condition is $(1-\phi)(a-2Q)-c=0$, hence,

$$Q^d = \frac{a}{2} - \frac{c}{2(1-\phi)}, \quad \text{and} \quad p^d = \frac{a}{2} + \frac{c}{2(1-\phi)}.$$

Now, industry profit is given by the sum

$$\begin{aligned} \pi^M + \pi^d &= \left[\phi Q^d p^d\right] + \left[(1-\phi)Q^d p^d - cQ^d\right] = \left(p^d - c\right) Q^d \\ &= \left(\frac{a}{2} + \frac{c - 2(1-\phi)c}{2(1-\phi)}\right)\left(\frac{a}{2} - \frac{c}{2(1-\phi)}\right) \\ &= \left(\frac{a-c}{2}\right)\left(\frac{a}{2} - \frac{c}{2(1-\phi)}\right) \\ &< \frac{(a-c)^2}{4}. \end{aligned}$$

Thus, this contract yields a lower profit than the industry's profit when the firms vertically integrate into a monopoly given in (14.24).

(c) Whereas the "profit sharing" contract analyzed in part (a) is optimal (i.e., maximizes industry's profit) it is rarely observed since it requires having the manufacturer monitoring the profit made by the dealer. This monitoring may be too costly for the manufacturer. In contrast, the per-unit contract analyzed in the text, despite being not optimal, does not require any monitoring since the dealer pays up front for each unit it acquires from the manufacturer.

5. We will demonstrate how a monopoly can benefit from establishing a trade-in procedure by constructing a simple model. Suppose that there are two consumers: one owns an old refrigerator and one that is a new buyer. The utility from using an old refrigerator is V_U whereas the utility from buying a new one is $\max\{V_N - p_N, 0\}$, where $V_N > V_U > 0$. We further *assume* that $V_N < 2V_U$.

   Single price policy: I will argue that the profit maximizing price is $p_N^* = V_N$, thereby selling only to the first-time buyer and earning a profit of $\pi^* = V_N$. To see this, note that the monopoly can set a lower price, $p^L = V_N - V_U$, thereby selling two units and earn a profit of $\pi^L = 2(V_N - V_U)$. However, by assumption, $\pi^L < \pi^*$.

   Trade-in price policy: We now show that by employing a trade-in price policy, the monopoly can price discriminate between new buyers and old-model owners. Suppose that the monopoly announces that current users can trade-in their old refrigerator for a new one for a price of $p_{TI} = V_N - V_U$, whereas the price with no trade-in is $p_N = V_N$. Note that under these prices, current users trade-in their old refrigerator for a new one, and also a new buyer buys a new one. Altogether, $\pi_{TI} = V_N - V_U + V_N = 2V_N - V_U > V_N = \pi^*$.

Chapter 15

Management, Compensation, and Regulation

Instructors who choose to teach from this chapter are urged to go over the principal-agent problem (section 15.1), which is developed three times in order to allow for a a gradual increase in the degree of difficulty. More precisely, the first subsection illustrates the problem with no uncertainty. A subsequent subsection introduces uncertainty with no risk aversion. The third subsection of introduces risk aversion implicitly by assuming asymmetric information. Altogether, these three subsections analyze the principal-agent problem in different degrees of complexity, thereby enabling the instructor to stop at the level that exceeds the students' ability.

Section 15.2 introduces a simple free-rider team effort problem. Section 15.3 introduces Fershtman-Judd model, which is recommended only if the students are capable of solving Cournot-type problems very easily. The remaining sections: executives' compensation (section 15.4) and the regulation of the firm (section 15.4) do fit the more advanced students.

Answers to Exercises

1. (a) Under collusion, the "social planner" chooses a common effort level for all scientists e that solves
$$\max_e \left(w - e^2\right) = \frac{V}{N} - e^2 = \frac{Ne}{N} = e - e^2,$$
yielding $e^* = 1/2$.

   (b) Each scientist i takes all other effort levels $(e_j,\ j \neq i)$ as given and solves
$$\max_{e_i} U_i = \frac{V}{N} - (e_i)^2 = \frac{\sum_{j \neq i} e_j + e_i}{N} - (e_i)^2,$$
yielding a NE where $e_i = 1/(2N)$ for all $i = 1, 2 \ldots N$.

   (c) Yes, since the optimal effort level is $e^* = 1/2$ which is independent of the number of scientists. In constrast, the NE effort levels decrease further below the optimal level when the number of scientists increases, reflecting a stronger free-rider effect.

2. (a) The manager of firm 1 chooses q_1 that solves
$$\max_{q_1} M_1 = 0.5[(a - q_1 - q_2)q_1 + (1 - c)q_1],$$
yielding a best-response function given by
$$q_1 = BR_1(q_2) = \frac{a - c + 1 - q_2}{2}.$$
The best response function of firm 2 (the firm that maximizes profit only) is given by $q_2 = BR_2(q_1) = 0.5(a - c - q_1)$. Therefore, the equilibrium output levels are given by
$$q_1^e = \frac{a - c + 2}{3}, \quad \text{and} \quad q_2^e = \frac{a - c - 1}{3}.$$

Clearly, $q_1^e > q_2^e$ confirming that the managerial compensating scheme of firm 1 is output increasing at the expense of the firm that maximizes profit only (firm 2).

(b) The owner of firm 1 earns $\pi_1^O = \pi_1 - M_1$. From the previous calculation we conclude that $p^e = (a - 2c - 1)/3$. Hence, $\pi_1^e = (a - c - 1)(a - c + 2)/9$. We now set μ_1 sufficiently small (as in [15.29]), so that we approximate the owner's profit by the firm's profit. Formally, $\pi_1^O = \pi_1$.

Finally, comparing the profit of the owner of firm 1 under the present managerial compensation scheme to her profit when this manager maximizes profit only (the simple Cournot profit given in (6.7)) yields that $\pi_1^O \geq \pi_1^c$ if and only if $a \geq 2 + c$. That is, the present compensation scheme dominates a simply Cournot behavior (given that the other firm behaves in a Cournot fashion) if the demand intercept is sufficiently higher than the unit production cost.

Chapter 16

Price Dispersion and Search Theory

Price dispersion (section 16.1) demonstrates that a discount store and an 'expensive' store can coexist in a market where consumers have different search costs (resulting, say, from different values of time). The analysis relies on a simple of location model. Thus, the students should find this section easy to follow. Search theory (section 16.2) is introduced with no calculus, so the analysis relies on simple probability arguments.

Answers to Exercises

1. (a) To see what happens, we look at the equilibrium prices when $L \to 0$. From (16.9) we see that

$$\lim_{L\to\infty} p^e_D = \lim_{L\to\infty} p^e_{ND} = 2\alpha H \quad \text{and} \quad \hat{s}^e \to 0.$$

Hence, when L becomes small (i.e., some consumers have a negligible search cost), both stores charge the same price, implying that there is no discount store.

(b) When $\alpha = 3/2$, the consumer indifferent between searching or buying at random is located at L. Hence, when $\alpha = 3/2$ all consumers buy at random. When $\alpha = 1$, $\hat{s}^e < L$ implying that the discount store a negative market share (which is ruled out by out assumption that $\alpha > 3/2$).

2. (a) Using (16.13), we have it that

s	0	1	2	3	4	5
$\bar{p}$	1	4.77	6.52	7.86	9.00	10.0

Solution-Table 16.1: Reservation price as a function of the search cost parameter

(b) Solution-table 16.1 reveals that for $s \geq 4$ the consumer reservation price is $\bar{p} = 9$, implying that the consumer will accept any price in the first store visit.

(c) Solution-table 16.1 reveals that $\bar{p} = 1$ when $s = 0$ (i.e., when search cost is zero).

(d) When $s = 0$,[1] the probability that the consumer finds the price higher than 1 (i.e., the reservation price) in a single store visit is $\sigma = 8/9$. The probability that prices exceed 1 in two store visits is $\sigma^2 = (8/9)^2$. Hence, the probability that this consumer finds prices exceed 1 in *all* store visits is $\lim_{T\to\infty}(8/9)^T = 0$.

Note also that this result can be inferred from (16.16) where the calculated expected number of stores visited is $\mu = 1/(1-\sigma) = 9 < +\infty$.

[1] The first printing of the first edition does not specify which search cost is assumed. This question should read, under the search cost you found in the previous subquestion, calculate....

Chapter 17

Miscellaneous Industries

The purpose of this chapter is to demonstrate to the student that there is no general model that can describe all industries; however, I also wish to demonstrate in this last chapter that the tools developed in this book are indispensable to the analysis of most industries, and are, therefore, worth learning.

This chapter discusses four types of industries that require special attention since (perhaps like any other industry) they do not fall in any category and therefore cannot be analyzed by simply picking one of the market structure analyzed earlier in the book. In future editions I will attempt to expand this part of the book by introducing at least three more industries: the health industry, the defense industry, and (you guessed, of course), the lawyers.

Answers to Exercises

1. From the definition, this technology (cost function) exhibits economies of scope if

$$\begin{aligned} TC(n,n,n) &= (n^\alpha + n^\alpha + n^\alpha)^\beta \\ &< (n^\alpha)^\beta + (n^\alpha)^\beta + (n^\alpha)^\beta = TC(n,0,0) + TC(0,n,0) + TC(0,0,n), \end{aligned}$$

   or,

$$(3n^\alpha)^\beta < 3n^{\alpha\beta}, \quad \text{therefore if} \quad 3^\beta < 3, \quad \text{hence if} \quad \beta < 1.$$

2. The main message here is that the introduction of a binding price cap by the regulator will cause the monopoly airline to reduce flight frequency (or any other quality aspect of the service).

   (a) Under the FC network, each route can be analyzed separately. Now, from the utility function of route 3 passengers (17.2), we see that the monopoly extracts maximum surplus when $p_3 = \delta + \sqrt{f_3}$. Since the airfare is regulated (i.e., $p_3 = \bar{p}_3$) we have it that the minimum frequency that the airline must provide on route 3 (at this airfare) is $\bar{f}_3 = (\bar{p}_3 - \delta)^2$. Clearly, as $\bar{p}_3$ decreases, the corresponding frequency $\bar{f}_3$ decreases.

   (b) Under the HS network, the flight frequency on route 3 is the (minimum) frequency on routes 1 and 2.[1] Hence, similar to (17.5), but with a given $\bar{p}_3$, the monopoly chooses $\tilde{f} \equiv \tilde{f}_1 = \tilde{f}_2$ that solves

$$\max_f \pi^h = n\bar{p}_3 + 2n\delta + 2n\sqrt{f} - 2cf,$$

[1] The first printing of the first edition contains a typo: Replace "...charges an airfare $\bar{f}_3$" with "airfare $\bar{p}_3$."

yielding a first order condition given by $0 = 2n/(2\sqrt{\tilde{f}}) - 2c$. Hence,[2]

$$\tilde{f} = \left(\frac{n}{2c}\right)^2 < \left(\frac{3n}{4c}\right)^2 = f^h.$$

Therefore, under the HS network, a price cap on route 3 will reduce the flight frequency on all routes.

3. The analysis here follows the same stages as in section 17.4, but it uses a different driving time function.[3]

   (a) If some people take the train and some drive a car, it must be that $L_T = L_C$. Thus,

   $$v + \phi = v(n_C)^3, \quad \text{implying that} \quad n_C^e = \left(\frac{v+\phi}{v}\right)^{1/3}.$$

   (b) The social planner chooses n_C^s that solves

   $$\min L^s = (N - n_c)(v+\phi) + n_C V(n_C)^3,$$

   yielding

   $$n_C^s = \left(\frac{v+\phi}{4v}\right)^{1/3} < n_C^e.$$

   Thus, the social planner would recommend a policy that would result in a reduction in the number of people who drive a car.

   (c) Let τ denote the highway toll on this route. Then, to set the toll that would generate the socially optimal number of highway users, the social planner sets τ^s that solves

   $$v + \phi = v(n_C^s)^3 + \tau = v\left(\frac{v+\phi}{4v}\right) + \tau,$$

   yielding that $\tau^s = 3(v+\phi)/4$.

---

[2]One has also to check that, in this equilibrium, the airfare on route 3, $\bar{p}_3$, is lower than the sum of the airfares on routes 1 and 2; that is, we need to verify that (or find the conditions under which) $\bar{p}_3 \le p_1 + p_2$. Otherwise, route 3 passengers can purchase two separate tickets and reach their destination at a lower airfare.

[3]The first printing of the first edition contains a typo. Simply set $t_C = (n_C)^3$ (the equation stated in this question equals to the *value* of the driving time, L_C and not t_C).