Constant Best-Response Functions: Interpreting Cournot

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Abstract

Following Amir and Grilo (1999), we characterize a class of demand functions that generate constant quantity best-response functions. Then, we draw the implications of constant best-response functions for the invariance of equilibrium outcomes with respect to the assumed market structure of quantity games. We then argue that, unlike the class of linear demand functions, this class of demand functions supports the pure interpretation Cournot's conjectures.

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1. Introduction

Perhaps the major problem in applying theoretical research on oligopoly theory is that equilibrium market outcomes tend to be highly sensitive to changing the order of decision makings of the firms in an industry, or the equilibrium concept. A natural question to ask is whether there exist demand functions in which the resulting market equilibrium outcomes are invariant with respect to the assumed market structure.

Amir and Grilo (1999, p.9) demonstrated the existence of a class of demand functions for which the resulting best-response functions are constant in quantity oligopoly games. The desired result in which the order of decisions in quantity oligopoly games does not alter the market outcome immediately follows.

From a theoretical point of view, the Cournot market structure may have two interpretations associated with two different assumptions concerning firms' expectations.[1] The widely-used (weak) interpretation is that a Cournot equilibrium is a Nash equilibrium, Nash (1950), in the sense that each firm expects the other firm to maintain a constant output level. Clearly, these expectations are consistent *only* in equilibrium, at least for the class of linear demand functions. However, a stronger interpretation of Cournot equilibrium is to assume that firms expect rival firms to have constant best-response functions. Thus, unlike the class of linear demand function, the class of demand functions identified in this paper is consistent with the strong interpretation of Cournot's conjectures.

This note is organized as follows. Section 2 identifies the class of demand functions yielding constant best-response functions. Section 3 demonstrates that the realization of output levels are invariant with respect Cournot and Stackelberg quantity setting market structures. Section 4 concludes with a general discussion and a demonstrating how in contrast to linear demand functions, this class of demand functions supports the pure interpretation of Cournot's conjectures.

[1]In fact, strategic games by themselves may have several interpretations; see for example Osborne and Rubinstein (1994, Section 2.1.2).

2. Demand Functions Yielding Constant Best-Response Functions

Consider a market for a homogeneous product with an aggregate downward sloping inverse demand function denoted by $p(Q)$, where Q denotes aggregate quantity demanded, and p the market price. We assume that $p(Q)$ is twice continuously differentiable with respect to Q.

There are N firms indexed by $i = 1, \dots, N$ ($N \geq 2$) which can costlessly produce this product. Let q_i ($q_i \geq 0$) denote the output of firm i, so $Q = \sum_{i=1}^{N} q_i$. Also, define

$$q \stackrel{\text{def}}{=} (q_1, \dots, q_N) \quad \text{and}$$
$$q_{\neg i} \stackrel{\text{def}}{=} (q_1, \dots, q_{i-1}, q_{i+1}, \dots, q_N).$$

Thus, the N-dimensional vector q is the list of the firms' output levels, whereas the $(N-1)$-dimensional vector $q_{\neg i}$ is the list of output levels of all firms except firm i. Finally, let $\pi_i(q) \stackrel{\text{def}}{=} p(\sum_{j=1}^{N} q_j)q_i$ denote the profit function of firm i, $i = 1, \dots, N$.

For given a $q_{\neg i}$, the set of maximizers is given by

$$\psi(q_{\neg i}) = \left\{ q_i \in \mathbb{R}_+ \middle| p\left(q_i + \sum_{j \neq i} q_j\right) q_i \geq p\left(x + \sum_{j \neq i} q_j\right) x, \quad \text{for all } x \in [0, \infty) \right\}.$$

We assume that $\psi(q_{\neg i}) \neq \emptyset$ for all $q_i \in R_+^{N-1}$. We call ψ the best-response correspondence. A best-response function, $R_i(q_{\neg i})$ is a selection out of $\psi(q_{\neg i})$; i.e., $R_i(q_{\neg i}) : R_+^{N-1} \longrightarrow \psi(q_{\neg i})$. A best-response function is said to be *constant* if there exists a constant $k_i \geq 0$ for which $R_i(q_{\neg i}) = k_i$ for all $q_{\neg i}$.

Following Amir and Grilo (1999, p.9), we state

Proposition 1

There exists a class of downward-sloping market demand functions for which the corresponding best-response functions are constant. Formally, this class of demand functions is given by

$$p(Q) = e^{-\frac{Q}{\beta}} = e^{-\frac{\sum_{i=1}^{N} q_i}{\beta}}, \quad \textit{for any} \quad \beta > 0. \tag{1}$$

Proof. We derive the best-response function of firm i, which chooses q_i that solves

$$\max_{q_i} \pi_i(q_i, q_{\neg i}) = q_i e^{-\frac{q_i + \sum_{j \neq i} q_j}{\beta}}.$$

The first-order condition is given by

$$0 = \frac{\partial \pi_i(q_i, q_{\neg i})}{\partial q_i} = \frac{(\beta - q_i)}{\beta} e^{-\frac{\sum_{j=1}^{N} q_j}{\beta}}.$$

Hence, $q_i = \beta$ is an extremum point. Since the second-order condition (evaluated at the extremum) is

$$\left. \frac{\partial^2 \pi_i(q_i, q_{\neg i})}{\partial (q_i)^2} \right|_{q_i=\beta} = \frac{(\beta - 2\beta)}{\beta^2} e^{-\frac{\sum_{j=1}^{N} q_j}{\beta}} < 0,$$

it follows that π_i attains a global maximum over $(0, \infty)$ at $q_i = \beta$. Observe that

$$\pi_i(q_1, \ldots, q_{i-1}, \beta, q_{i+1}, \ldots, q_N) = \beta e^{-\frac{\beta + \sum_{j \neq i} q_j}{\beta}} > 0.$$

Finally, since $\pi_i(0, q_{\neg i}) = 0$, π_i attains a global maximum over $[0, \infty)$ at $q_i = \beta$. It follows that $\psi(q_{\neg i}) \neq \emptyset$, and in fact $\psi(q_{\neg i}) = \{\beta\}$. Therefore, the best-response correspondence is a function. □

3. Equilibrium Oligopoly Market Outcomes

We now solve for the equilibrium market outcomes under Cournot and Stackelberg market structures.

A Cournot-Nash equilibrium, Cournot (1838), is an outcome $q^c \in \mathbb{R}_+^N$ such that $R_i(q_{\neg i}^c) = q_i^c$ for all $i = 1, \ldots, N$. Clearly, in this environment, the unique Nash-Cournot equilibrium is $q_i^c = \beta$ and the aggregate output level is $Q = N\beta$.

A Stackelberg equilibrium (in which firm 1 is a leader), Stackelberg (1934), is the outcome $q^s \in \mathbb{R}_+^N$ which is obtained by solving

$$\begin{aligned} \max_{q_1} \quad & \pi_1 (q_1, q_{\neg 1}) \\ \textbf{s.t.} \quad & q_j = R_j(q_{\neg j}) \quad \text{for all } j \geq 2. \end{aligned} \tag{2}$$

That is, the leader solves for the followers' best-response functions, and chooses its output level accordingly. Followers behave in a Cournot fashion. We deliberately refrained from defining Stackelberg market structure that employs a specific extensive-form game for the following reasons:

(a) Every extensive-form game has unique normal-form representation, however the converse does not hold.

(b) Using normal-form games is aligned with the equilibrium notion, that is the Nash equilibrium. In contrast, the mere specification of an extensive-form game suggests that the additional structure is meaningful and therefore calls for a refinement of the equilibrium notion, and this may yield different solutions.

(c) The set of Nash equilibria of a given extensive-form game coincides with the set of Nash equilibria of the associated reduced normal-form game.

(d) Most importantly, from the empirical point of view, applied economists do not observe decision processes.

Proposition 2

The realization of output levels are invariant with respect to the choice of Cournot and Stackelberg market structures.

It is interesting to point out that for this particular inverse-demand function, each firm can consider itself to be a Stackelberg leader in the sense that it solve the maximization problem (2), while firms' conjectures maintain mutual consistency. This invalidates a common perception that in a Stackelberg market structure when there are two firms, there is no equilibrium when both firms view themselves as leaders [e.g., Kreps, p.330]. Moreover, our result here is related to the endogenous-timing papers such as Hamilton and Slutsky (1990) and Amir and Grilo (1999) which obtain in more general setting the possibility that both firms will choose their output level at the same time (simultaneous game). Here we demonstrate the possibility of having a complete invariance with respect to the order of moves in the absence of production costs is not mentioned in this literature.[2]

[2]The issue of production cost turns out to be important since it is clear that for sufficiently-high production costs at least one firm will not produce thereby making the market outcome invariant with respect to the order of moves.

4. Interpreting Cournot: A Discussion

From a theoretical point of view, we would like to point out the following.

(a) Cournot (1927) has never suggested the widely-used class of linear demand functions as an example for his model. The class of linear demand functions belongs to a modern interpretation of his model. In fact, it seems as if Cournot left it vague which class of demand functions are consistent with his model.

(b) Unlike the class of linear demand functions, the class of demand functions identified in this note is consistent with the strong interpretation of Cournot conjectures in the sense that firms' correctly believe that rival firms will not deviate from their output level regardless of their actions. This interpretation does *not* hold for the class of linear demand functions.

(c) One *cannot* argue that the widely-used class linear demand functions is more general than the class identified in this note. In fact, as we argue below, we believe that the present class of demand functions better suits econometric fitting.

(d) It is indeed possible, that Cournot himself thought about constant-best response functions. Our interpretation of Cournot is based on Cournot's system of two equations given in Cournot (1927, Ch.7, p.66), which can be written using our notation and for $N = 2$ as:

$$\frac{\partial \pi_1(q_1, q_2)}{\partial q_1} = 0 \implies p(\beta + q_2) + \frac{\mathrm{d}p(Q)}{\mathrm{d}Q}\frac{\partial Q}{\partial q_1}\beta \equiv 0, \quad \forall q_2 \tag{3}$$

$$\frac{\partial \pi_2(q_1, q_2)}{\partial q_2} = 0 \implies p(q_1 + \beta) + \frac{\mathrm{d}p(Q)}{\mathrm{d}Q}\frac{\partial Q}{\partial q_2}\beta \equiv 0, \quad \forall q_1. \tag{4}$$

Equation (3) reflects what firm 2 conjectures about firm 1, namely that firm 1's optimization *always* yields $q_1 = \beta$ *regardless* of the level of output set by firm 2. Similarly, equation (4) reflects what firm 1 conjectures about firm 2, namely that firm 2's optimization *always* yields $q_2 = \beta$ *regardless* of the level of output set by firm 1.

> Therefore, our interpretation of Cournot's conjecture is that equations (3) and (4) holds as *identities* independently of each other. In contrast, the common interpretation is to require that this system of equations holds in equilibrium in which case the Cournot conjecture about rival firms holding their output constant is inconsistent with firms' having downward sloping best-response functions.

Finally, various authors [e.g., Bergstrom and Varian (1985)] have attempted to facilitate empirical research in oligopoly markets using simple Cournot market structures. The class of demand functions characterized in this note has significant implications for empirical research. The reason for this is that models in which econometricians can fit the data into a demand function in the class identified by (1) can have strong prediction power because the market outcome is invariant with respect to the assumed market structure. This means that in these markets, the applied economists does not have to search in the class of sequential dynamic games in order to find a particular ad-hoc order of moves which would fit the data best.

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